The article written by Debbie Morgan entitled ‘Five Big Ideas’, published in MT227, struck a chord with us. Like Debbie and her colleagues, we at the University of Brighton have structured our Primary Mathematics Specialist Teacher Programme around ‘big ideas’ in mathematics. We sought an approach that reflected our belief that mathematics learning is achievable for all children, and that it can be more accessible than children often experience. Structuring our programme around such ideas, we argue, deepens teachers’ subject knowledge, supports the development of pedagogies that emphasise mathematical connections, and ultimately enables the development of relational understanding (Skemp, 1989) in both teachers and children.

However, within our primary mathematics tutor team, it has been our discussion of big ideas that has been the most significant professional learning experience for us. What do we mean by a big idea? How big does an idea have to be to make it onto the list? Why, and how, can such ideas be relevant in the everyday planning and teaching of primary mathematics? Is there already an agreed list out there somewhere? Is the process of engaging in the thinking more important than the specific content of any particular list produced?

A brief exploration reveals previous productive thinking in this area. In 2008, ACME produced a discussion paper to support the Rose Review of the primary curriculum and identified big ideas including: place value and the number system, conservation of number and measures, equivalence relations, dimensionality. In a much earlier edition of MT, Geoff Faux (1998, MT163) identified a set of 6 big ideas: numbers are ordered and well structured, mathematics is shot through with infinity, a lot for a little, equivalence, inverse, transformation.

However any list presented represents the product of its author’s thinking, not necessarily the process and it was clear to us that we needed to engage with this thinking process ourselves. We needed first to rationalise what we meant by a big idea and through this to identify a set of such ideas that would support the practice of the experienced primary teachers with whom we were working. We define a big idea as a broad, underpinning and recurring theme applicable at any level, and across all branches of mathematics. Our premise was that continued focus on such a set of ideas would enable teachers and children to develop connection and coherence in a curriculum which comprises micro-level objectives. This is supported by a range of research (e.g. Ritchhart, 1999, Smith and Girod, 2003) and essentially enables the question ‘but what, mathematically, is this really all about?’ to be addressed.

It became increasingly clear to us that we needed to engage MaST teachers in a similar process of questioning and reflection on the value and composition of a list of big ideas. Firstly, we have tried to enable the MaST teachers to construct their own understanding of the mathematical relevance of individual big ideas. We have presented our rationale for the selection of each idea, and asked them to explore and reflect on application within their own practice. Secondly, we have sought to engage MaST teachers in a more holistic and ongoing dialogue about the value of a big ideas approach.

Discussion of these ideas has continued throughout the development and leading of this CPD programme. The list has been refined and re-ordered over-time. Our current list, in the order in which we have most recently presented it with no hierarchy implied, is outlined below:

**Pattern and structure**

The search for pattern for some people defines mathematics. We have underpinned this pursuit with developing an understanding of the mathematical structures that give rise to patterns. For example, our number system has a base ten structure and this gives rise to the particular scaling pattern of successive columns in the recording of digits. It also gives rise to the patterns we see in multiplication tables. For example overleaf, multiples of 6, arranged in rows of width 10 reveal why some multiples of 6 are necessarily partitioned into 4 + 2 giving rise to the patterns in the units digit in the x6 table.
Representation

The importance of representing mathematical thinking is a significant feature of primary mathematics lessons. We have sought to highlight the need to represent mathematical ideas, particularly through the effective use of Bruner’s (1967) enactive mode of representation, in all aspects of primary mathematics and across all ranges of age and attainment. The use of the enactive mode has enabled teachers to see things differently, and many have noted that this had deepened their conceptual understanding. This big idea enables the expression of other big ideas, for example, the use of representation enables pattern to emerge, and is supportive of prediction and establishment of generality. In this addition pyramid, representation supports understanding of why the order of numbers on the bottom row impacts on the size and properties of the total.

Logic

We seek to develop teachers’ understanding and awareness of logic through a focus on attributes and classification, problem solving, and the use of language constructs that support the expression of logical statements. For example: using a loop of elastic and two pairs of hands, create a regular quadrilateral shape. How do you know it is a regular quadrilateral? What is the smallest number of properties that you would need to measure to ensure it is a regular quadrilateral? Be precise: which angles? Which lengths?

Equivalence

Here we focus on developing an understanding of how and why mathematical concepts and relationships can be expressed as equivalences. This builds on the big idea of logic, and enables the development of flexibility in the way that we see, understand, represent, and articulate equivalences in number and shape. Throughout the exploration of this big idea, we focus on the nature of the relationships that produce equivalences. For example, we explore solutions to $6 \times \square = 2 \times \square$ using enactive representations such as a 2-pan balance and Numicon and focus on the relationships between the numbers that satisfy the equivalence.

Divergent thinking

This encompasses the development of approaches to solve mathematical problems. Having originally named this big idea ‘problem solving’ we subsequently adopted the term divergent thinking to better describe the processes involved, for example forming and testing conjectures, and perhaps to distinguish this from ‘stepwise’ routines that are commonly applied to word problems. In naming this big idea, we have sought to provoke reflection and further thought rather than assume a shared and common understanding.

Comparison: Proportionality

Within the primary phase multiplicative comparison appears to be a mathematical concept that is less-well developed than additive comparison. We have therefore focused our big idea of comparison on proportional reasoning that arises from multiplicative comparison.
in her earlier article (2012, MT227). We also recognise significant overlaps within our big ideas; in seeking to expose connections between mathematical ideas, we find the interconnectedness of our list to be reassuring. At this stage we conclude that there is more than one way to construct such a list. For us, the most productive elements have been the discussion about the mathematical validity and significance of the big ideas and the connections between them. One notable feature emerging from developing and refining each aspect of this list as we have taught the course to the four cohorts has been that our list increasingly emphasises mathematical behaviours. We have come to the philosophical view that these behaviours are more than a route to mathematical knowledge; they constitute mathematics itself, thus we understand mathematics as human activity. Moreover, and happily, they reflect much of what is frequently asserted to be universal human behaviour — the search for pattern, the use of logic, prediction, and generality (Mason and Johnston-Wilder, 2004). This connection between human disposition and mathematical behaviour is central to our approach. If a focus on these ideas appeals to the way that the human brain operates then mathematics is a subject for the many, not the few. This is a heartening thought; all children can be mathematical and teachers can develop their capacity to teach it.

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References:
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