Ian Thompson suggests that ‘evidence-based’ should refer to all the evidence when it comes to mathematics education.

Ofsted was asked by ministers to provide evidence of effective practice in the teaching of early arithmetic. This led to the publication of *Good practice in primary mathematics: evidence from 20 successful schools* in November 2011. The schools visited - 10 maintained, and 10 independent – all had a strong track record of high achievement. The progress of those pupils in maintained schools in Key Stage 2 mathematics tests had been significantly above the national average for the last four years.

The document includes short chapters on:

- the timing of the teaching of formal methods for column calculation
- what schools do to secure successful progression to such procedures
- the place of problem solving
- identifying and overcoming calculation difficulties
- school calculation policies
- teacher subject expertise and continuing professional development (CPD)
- working with parents/carers.

In this article, however, I plan to focus on those sections that deal with the actual content of the arithmetic curriculum the children were taught; the specific procedures they used as a result of this teaching; and those they described in discussion with Ofsted inspectors. I do this as an experienced researcher of children’s mental, and written calculation methods, who feels that permeating this document is an undercurrent of negativity whenever phrases such as ‘informal procedures’, ‘expanded algorithms’ and ‘interim methods’ are used. These interim methods would include the informal written calculation strategies known as front-end addition, subtraction by complementary addition, grid multiplication and division by chunking - procedures that, interestingly, I first came across in *Mathematics 5-11: a Handbook of Suggestions* (DES 1979: 29-33), an HMI publication in their ‘Discussion Matters’ series.

It is also interesting that Ofsted/HMI appear to be contradicting their own advice given in earlier publications. For example:

*In number, pupils are not always supported well enough in the use of informal methods of calculation when they cannot cope successfully with standard written methods.* (Ofsted 2002a: 2);

*The most successful teachers balanced the requirements for pupils to record their calculations in a standard form with the opportunity to write and record their work in their own way.* (Ofsted 1993: 10);

*However, at Key Stage 2 they [teachers] often overlook the importance of linking pupils’ mental strategies to the introduction of expanded and compact written method.* (Ofsted 2002b: 3).

*There is considerable evidence to indicate that many ‘imposed methods’, on which much time is spent in schools, are often quickly forgotten, and that pupils or adults revert to their own methods which they understand and in which they are more confident.* (DES 1985: 4);

*By practising only one method at a time, pupils do not gain the confidence and intellectual flexibility they need.* (Ofsted 2008: 8).

*In the classes where standards were low, it was frequently the over-emphasis on written recording and computation.* (Ofsted 1993: 10).

I have critiqued other aspects of this report - including some of the actual mathematics! – elsewhere, (Thompson 2012), and so will focus here on the algorithm that receives the worst press in the document, namely the written division method known as ‘chunking’. The procedure is mentioned in at least eight places in the document, and never in a positive light as are other division methods. This is despite the fact that the report mentions problems with all three main division algorithms, saying that:

*…many (maintained schools) expressed...*
unease about the effectiveness of the different methods, principally ‘chunking’, short division, and the traditional long division algorithm. (p. 8).

Of course, it is clearly just a coincidence that the Schools Minister, Nick Gibb, is not particularly enamoured of informal methods - chunking in particular!

**The ‘chunking’ algorithm**

The following section comprises several statements from the document about ‘chunking’ followed by a few thoughts about these statements.

1. Nearly half of the schools in the survey do not teach ‘chunking’ as a strategy for division. They explained that it confuses pupils, particularly those who are low attaining. (p. 16).

Let us see what insights research - national and international – has to offer concerning this declared ‘confusion? Anghileri et al. (2002) compared the results of 276 English Year 5 children from ten schools with 259 Dutch children from similar backgrounds on a ten question division test. In January the English success rate was 38% and the Dutch was 47%. However, on the same ten problems the success rate rose in June to 44% for the English children and to 68% for the Dutch children. The researchers concluded that:

The traditional algorithm was widely used by the English pupils who showed less progress than the Dutch pupils who used written methods based explicitly on repeated subtraction (p.1).

In 2003 Anghileri (2006) compared the results of the English students in the earlier study with the written responses of 308 Year 5 students to the same ten division problems, and found that for the 2-digit by 2-digit division questions, the success rate had improved from 51.3% to 60.1%. In 1998 no child had used the chunking algorithm, whereas in 2003 the same percentage used it as used the traditional algorithm. The success rates differed by 23% in favour of chunking (55% to 32%).

Unfortunately the document fails to mention the potential ‘confusion’ caused by the long division algorithm. Ofsted has chosen to ignore all the research from the 1980s and 1990s on the learning, mis-learning, and forgetting of such algorithms. To give just one example, Hart (1981: 212), when considering the implications of the extensive research of the Concepts in Secondary Mathematics and Science (CSMS) team, concludes:

The interviews on every topic showed that children for the most part did not use teacher-taught algorithms... To a great extent children adapt the algorithms they are taught or replace them by their own methods...

2. Because they do not spot the larger multiples of the divisor, they tend to work repeatedly in smaller steps of 10 times and 1 times the divisor. (p. 16).

This, of course, depends on how much work the children have done on multiples. One useful strategy to help them develop confidence in this area, using just the basic skills of doubling, halving and multiplying by ten is to work on what I call ‘partial multiple tables’. For example, if dividing by 36 the basic table looks like this:

| 1  | → | 36 |
| 2  | → | 72 |
| 4  | → | 144 |
| 10 | → | 360 |
| 5  | → | 180 |

Figure. 1

Three or six 36s can be found by addition, and if the dividend is a larger number, the table can be easily extended by multiplying 72 by 10 to find 20 thirty-sixes, etc. - see (Thompson 2005).

3. It is not clear why pupils who are able to understand the short division method seem to become confused when trying to record the same steps in the formal algorithm... (p. 28).

This sentence follows a description of a school that teaches short division with 2-digit divisors. If the sentence is referring to such division, then the obvious explanation is that children do not usually learn the multiplication tables of 2-digit numbers. There also seems to be some confusion about the word ‘understanding’, in that there is a very great difference between being able to correctly execute the short division algorithm and actually understanding it. Take the calculation 104 ÷ 8 where the spiel goes something like: “8 into 1 doesn’t go [although it actually does if the one has its real value of 100]... 8 into 10 goes one remainder 2... [8 into 10 what? One what?].... Carry the 2... 8 into 24 goes 3.” It is very difficult to
give meaning to this procedure – it just has to be learned – and forgotten, as much research in the 1980s showed us. In my own case I successfully learned subtraction by ‘equal additions’ in primary school, but never came to terms with the apparent unfairness of borrowing from one place - thin air? - and paying back to another!

4. A further issue is that the mathematical thinking behind the method of short division, which most pupils master, is different from the thinking behind ‘chunking’; one method does not lead into or support the other. (p. 16).

So, Ofsted appears to be saying that by teaching the ‘bus-shelter’ short division algorithm you end up with - most? - children unable to use it for dividing by a 2-digit number; unable to progress to chunking because the ‘thinking’ behind it is different; and, because the thinking behind long division is also different, are just as likely to be unsuccessful using this algorithm. A simple solution to this problem might be to teach ‘short division’ the way the Framework (DfEE 1999) introduced it, i.e. as single-digit chunking. This is then more likely ‘to lead into, or support, the other (method)’. For a more detailed critique of short division see Are we selling the children short? (Thompson 2011).

Also, in offering an explanation for the improvement in English children’s results in the research described above Anghileri (2006: 377) argues that:

> Overall, the results show a shift from extensive use of the traditional algorithm in 1998 to more use of informal methods in 2003 and different written methods, particularly in problems with 2-digit divisors where the short division algorithm failed in the 1998 study.

5. Schools that teach the ‘chunking’ method for long division acknowledge that some pupils have difficulty spotting large multiples and that errors creep in with the repeated subtractions. (p. 28).

Even as an enthusiast for ‘chunking’ I have been critical of the emphasis given to ‘subtractive chunking’. The argument often provided in favour of the procedure concerns ‘choice’: the most able child can work with large chunks whereas the less able child can use small ones. The only problem is that for the less able, this leads to more subtractions – an operation that the child is probably not too proficient at in the first place. For this reason I have argued for ‘additive chunking’ or ‘chunking up’ (Thompson, 2005).

A Mathematics Specialist Teacher (MaST) writes in Edge Hill University’s Masterclass magazine (Parkes 2011) about teaching his Year 6 class the ‘chunking up’ method using the ‘partial multiples table’ idea illustrated in Figure 1 on page 46. He found a substantial improvement in his pupils’ performance on SAT division questions and also in their confidence. Writing in Primary Mathematics (Bradford 2011), another MaST teacher reports on a small-scale research project in which Year 3 children were challenged to use their own written methods to solve simple two-digit by one-digit divisions. Probably because the teacher had emphasised division by grouping, the vast majority of children solved the problems using multiples of the divisor in a manner similar to that illustrated in Figure 2.

Figure 2

This and many other examples in the article suggest that working forwards is much easier than working backwards for at least two reasons:

- working backwards would give the sequence 31, 27, 23, 19, 15, 11, 7, 3 – an unfamiliar sequence compared to the multiples of four;
- in Figure 2 it is straightforward to count the number of fours in 31 and read off the remainder 3, whereas it is not as obvious in the subtraction sequence. Do you start at 31 when you haven’t subtracted a four yet? Do you include the 3 which you might feel you are counting twice if you also count it as the remainder?

This method is obviously more useful when dividing numbers of any size by a single-digit number, however, making use of a ‘partial multiples table - Figure 1 - enables the procedure to be used for dividing by two, or three-digit numbers. Bradford’s conclusion is that:

> This work appears to offer ‘practical’ evidence to back up Thompson’s (2005) theoretical arguments concerning ‘complementary multiplication’. (p13).
On the same theme, Borthwick and Harcourt-Heath (2007) analysed the answers of 995 children on four Year 5 Qualifications and Curriculum Authority (QCA) test questions. They repeated the analysis four years later (Borthwick and Harcourt-Heath 2010) using the same questions with a cohort of 1068 Y5 children from the same schools. On the second occasion 28 per cent of the children achieved the correct answer on the division question – an increase of 7 per cent over the previous cohort. Also, the number of children successfully using the ‘chunking up’ strategy was greater by almost 50 per cent than the number of successful children using any other strategy.

Concluding remarks
Whatever your views on written division methods, I think it is difficult to disagree with Anghileri (2006: 278) when she argues that:

What is desirable is progressive, structuring of a written record that builds on pupils’ informal approaches and... is in tune with pupils’ established ideas of division.

We know that this government is enthusiastic about ‘evidence-based practice’, and that this document has been commissioned in order to gather such evidence. However, it is to be hoped that when the final version of the National Curriculum appears it is underpinned by, not just reports such as this, but by all the existing research on the teaching of written calculation methods.

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