I push very, very hard number lines; that they [the students] should draw number lines, have pictures and so on.

My interest in the use of the number line as a powerful image of the real numbers, and how this is introduced in the early years, comes from my own experience as a mathematics learner. The quote above is an extract from a study I was involved in (Iannone, 2004). The person speaking is a first year analysis university lecturer reflecting on issues relating to students’ difficulties with limits. I myself have extensively used the number line during my university studies in mathematics as a tool that allowed me to visualise less intuitive properties of the real numbers. When I started working on a research project with Y1 children I became interested in exploring how the number line is introduced and used by primary teachers and their pupils. Would this reveal the beginning of the construction of a mental image that the children will be able to build on and carry along for years to come?

**Background**

Firstly, I was interested to understand if the number line was used so extensively in the primary classroom, and how the teachers themselves saw its use. From the national numeracy strategy (NNS) Framework I learned that

Beside the board, each class should have a large, long number line for teaching purposes, perhaps below the board . . . For Reception and Y1, number tracks with the spaces numbered to 20, rather than number lines with the points numbered, are helpful, including those made of carpet tiles.

(NNS Framework, p29)

From the above recommendation we can see that the NNS strongly advises the use of the number line, but it suggests the use of number tracks for the early years. The Framework does not point to any real difference between number lines and number tracks, and seems to imply that these are different representations of the same concept. Certainly, in all the classes I have observed, number tracks were used extensively, but not so number lines. There are, I believe, at least two reasons why the NNS supports the use of number lines (and/or number tracks) in the early years. The first, as Millet et al point out (2005), is that understanding what constitutes an acceptable level of numeracy for young pupils depends on the socio-cultural context. Hence policy makers in England decided that

The aspect of numeracy to be newly emphasised at primary level should be proficiency in a culturally neutral context-free set of number skills, underpinned by abstract visual models, such as the number line.

(Millet et al, pXXIII)

Moreover, educators and policy makers in the UK have been influenced by the large body of research carried out in Holland – known as Realistic Mathematics Education (RME). In the Netherlands, RME has inspired curriculum changes and teaching practice for the past 25 years. For a recent example of the use that RME suggests for the empty number line and how this compares to British views on mental arithmetic see Beishuizen,
1998. Dutch researchers pointed to the empty number line as a powerful tool for strengthening pupil’s mental arithmetic skills. On the empty number line children are encouraged to perform sums of two or three digit numbers with the method illustrated in figure 1. This will be familiar to anyone who has recently taught mathematics in primary schools in the UK.

The study

The main prompt for reflection on the use of the number line in the early years was a study we carried out with Dr A Cockburn at the University of East Anglia and funded by the Economic and Social Research Council (RES-000-22-0851). It was structured around five multiple case studies of experienced teachers and their Y1 classrooms. Among the data collected were interviews with the teachers, observations in the classroom and videos of lessons. The two main aims of this study were

- to investigate the presence of conceptual mathematical thinking with pupils as young as five years of age, and
- to investigate the engagement that very young pupils have with mathematics.

The reflections about the use of the number line arose in the context of the first of the above aims.

What did I observe?

The observation period was an extraordinary opportunity to become familiar with the life in the primary classroom. We observed the consistent use of number tracks, number snakes and number washing lines – but number lines were very seldom used. Only one teacher in the study consistently used a number line as shown in figure 2.

Sometimes this number line would go up to 10, or 20, or more, sometimes it would be very short, just enough to deal with the sum the pupils were looking at. In all the other classrooms I visited the teachers were using number tracks (see figure 3) number snakes, number washing-lines, etc.

These are all very useful prompts that help pupils with early numeracy and they are not the only ones available. In MT195 (Harrison, 2006) I read of a recent re-interpretation of the number strip called the Numdrum. This suggests pupils visualise numbers from 0 to and beyond 100 as a continuous helix, hence overcoming the problems usually found with number squares. (How do we carry on counting once a line is finished? What comes after 100?) However, all these models have in common that they convey the number system as a discrete set.

When I asked the teachers in the project their views on the use of number tracks, I received a variety of answers:

- number tracks are consistent with number squares;
- number tracks are a useful reminder of what the numbers look like (in the same way that alphabet tracks are);
- number tracks are invaluable help for counting;
- number tracks help with ordering the non-negative integer numbers.

These are all very valid reasons to use number tracks; however, number tracks account only for cardinal numbers. By cardinal numbers we mean numbers that denote the number of elements in a set, hence numbers that are immediately linked to counting. Cardinal numbers are not immediately useful for talking about fractions. More generally, number tracks do not account for ordinal numbers.

As we often intuitively define the real numbers as numbers that are in one-to-one correspondence with the points on an infinite line – the number line – there seems to be a case for introducing this visual image right from the early years.
Introducing numbers in a different way

A different way of introducing the number line and understanding the way we teach numbers in the early years is explained in the work of the American educator Jean Schmittau (Schmittau, 2003). She observes that in our primary classroom we teach numbers through and for counting, which is also how the number system historically evolved. A lot of emphasis is put on non-negative integer numbers as tools for counting objects. However, only the non-negative integer numbers can be associated with counting. In this way, when other numbers within the real numbers are introduced (such as fractions in Y2), the child’s understanding about where numbers come from has to be radically changed.

Building on the work of the Russian educator V V Davydov, Schmittau proposes to introduce numbers through measuring. If numbers are introduced through measuring, then potentially all real numbers can be accounted for and there will be no adjustment required later on to accommodate fractions or irrational numbers. In one of her papers she reports on one of her teaching experiments with five- and six-year-old pupils. (Note that this is the age when pupils start school in the American system.) Children are asked to measure a given length from a smaller given unit. This might fit an integer number of times in the length, say for example, three times, as in figure 4.

This will familiarise children with the integer positive numbers. If the unit does not fit an integer number of times in the given length, the children are asked to see how many times the remainder will fit into the unit length, as in figure 5.

In this case the measure of the segment is three units and one half of a unit, hence $3\frac{1}{2}$. And so on.

The mathematics beyond this concept of numbers is of course continued fractions, as we can see below. For every fraction $\frac{P}{Q}$ ($P$ and $Q$ are integer, positive numbers) we can find integer numbers $a, b, c, \ldots$ so that

$$\frac{P}{Q} = a + \frac{1}{b + \frac{1}{c + \frac{1}{d + \ldots}}}$$

This is called the continued fraction expression for $\frac{P}{Q}$. It is easy to see that the integer $a$ in the expression above counts the number of times $Q$ fits into $P$ ($a$ is 3 in figure 5, where $P$ is the given length and $Q$ is the given unit) and the integer $b$ counts the number of times a remainder fits into $Q$ (in figure 5, $b$ is 2). If $\frac{P}{Q}$ is less than 1, then the first number, $a$, will be 0. In this way, every fraction can be obtained after a finite number of steps. If this process has no end, if at no stage we can fit a remainder into the preceding one an integer number of times, then we have obtained an irrational number (in fact, irrational numbers cannot be represented as fractions). In this way all real numbers could be constructed and accounted for. This way of introducing numbers is of course presented gradually, with very young children only engaged with measures that produce an integer number and progressively during the school years refined to measures that produce fractions and so on.

A suggestion for an activity with Y1 children

Although the way of introducing numbers in the early years illustrated in Schmittau’s work is very different from the way suggested by the NNS, perhaps it would be possible to incorporate some elements of the former when engaging young children with measures. For example, as part of the activities suggested by the NNS, Y1 pupils are asked to measure a given length in a chosen non-standard unit. A typical hands-on activity I have observed often in the classroom is measuring how far a toy car travels when pushed down a slope using building blocks as unit.

After the pupils have built the slope and pushed
I would like to thank my colleague, experienced teacher and friend Harry Grainger, for pointing me towards Schmittau’s work and for patiently listening to my ramblings about the number line.

References

The National Curriculum for Mathematics sets the expectation that Year 2 children will be able to measure length using blocks. Once they have had some experience of this in the early years, I suppose that the answer is that if we want only the number tracks are predominately used in the early years, I suppose that the answer is that if we want to introduce non-negative integer numbers, those models are perfectly adequate and indeed easier to use, than the number line. However, I hope I have made the case for introducing children from an early age to a more general concept of number that will not need to be changed along their time as learners of mathematics. The number line would then be the ideal visual image to underpin understanding of this more general concept of numbers, namely the real numbers.

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“overheard”

Year 2 children were given the problem that ‘The answer is 10’. We tried a few examples together and they all set to work with enthusiasm. Most children started with addition, mainly adding only two numbers before getting adventurous and trying three or more. After a while they changed to subtraction and towards the end of our session Aaron turned to me with great satisfaction and said, “Do you know that if you do adding there are only so many sums you can do, but if you do take-aways you could keep going forever!”

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“overheard”

In a Reception class we were all playing a car racing game. We had a track for twelve cars and two dice. Twelve children were responsible for moving one of the cars. We threw the two dice, added the spots and then the person in charge of that car number moved it forward one space. After about ten throws Emily burst into tears.
“IT’s not fair”, she said, “I’m never going to move”.
Yes, she was in charge of car number one!
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