MODELLING IN THE MATHEMATICS CLASSROOM

Clare Lee

This article describes my thinking about ‘modelling’ as a teaching technique in a mathematics classroom; it is not about mathematical modelling. I think this modelling process is very important as a way to help pupils learn and in this article I try to say why. I find it very difficult to define what I mean by using a model when teaching, although I, along with others, cheerfully say I used a ‘model’ here or they ‘modelled’ that process there. If modelling is a useful process in a classroom why is it useful? What is it useful for? What can you achieve by using a model? What is a model when used in a classroom anyway? This is my voyage of exploration.

I am currently trying to research what went on in my classroom as I concentrated on language issues in the learning of mathematics. The classroom that I am investigating is my Year 9 class in a city comprehensive school. Year 9 were grouped according to the abilities they had shown in a test at the end of Year 8 and this was one of two middle groups out of four in this half of the year. They had little belief in their own abilities to do mathematics and a strong belief that mathematics was boring. The group consisted of twenty ordinary pupils from this type of school, some with English as a second language, some with challenging behaviour, but most ‘ordinary’ Year 9 pupils.

I was looking closely at some strategies that I had used in this classroom, trying to analyse what was going on. I used the word ‘modelling’ as a description of what I was doing. It was only when I had ascribed this term to several different situations that I realised I was describing something that was not simple; I needed to know more about what I meant when I said ‘modelling’.

This point was brought home to me by two incidents in the last week. In one, a teacher said she had to ‘model’ groupwork for her pupils, because they had little idea what that meant; in the other, I was told that two teachers ‘modelled’ problem solving for their pupils. Neither of these incidents are ones that I would immediately have thought of as models, for reasons I will explain further on. However, I could see that both incidents share something fundamental that makes them ‘models’ and not examples, cases, illustrations, images or anything else that we use in classrooms. What is this ‘something’ and why is it important?

In mathematics classrooms we commonly present examples to show our pupils how to use a particular piece of mathematics. These examples are carefully chosen to illustrate points that may come up and help pupils to grasp some of the nuances in using a particular skill in mathematics. I do not think this is the same as a model. Examples that may be used when teaching about Pythagoras’ Theorem might show finding the hypotenuse, then finding one of the shorter lines and then possibly finding the height of an isosceles triangle. They show learners a way that they can use Pythagoras’ Theorem: frequently this would be referred to as the way to use Pythagoras’ Theorem. In some ways this is restrictive. An example has some element of authority: this is the best way to do it. Although as a mathematician I welcome a good example to remind me what to do when I am using an approach I have not used for a long time, I come to examples with a different attitude from that of many school pupils learning mathematics. Examples tend to be the efficient way to approach a problem: they will be written out neatly showing all the logical steps. They could be thought of as excluding, since only those ‘literate’ in the grammar of mathematics will have access to examples. Certainly, the examples in a textbook were not accessible to those pupils whose learning I was considering.

Examples carry all sorts of connotations. A pupil might say, ‘If the example shows the right way to do it and I cannot make head or tail of the example then I cannot do it.’ They do not say, as I might want them to say, ‘I don’t understand that example, give me another one.’ Examples written in textbooks or written on the board can be very powerful objects. This seems to be one of the features of what was being described in the two incidents quoted earlier, and that may inhibit their ability to be ‘good’ models. Teachers ‘modelling’ what to do carries the same power relationship problems as examples. If the pupils watching teachers solving problems assume that their example should be followed reli-
giously then it may not help these pupils with their problem solving. The teachers have the authority here and therefore it may be hard for them to signify that this is only one way to problem solve, even when they are trying to help the pupils to get an idea of what it means to problem solve, rather than showing them the one way to do it. In contrast, one feature of the modelling process is that it is intended to give an idea of the quality of a way of working, rather than a royal road to follow.

So when I refer to ‘models’ I do not mean examples. Yet, I am not sure that I am any nearer defining what I, and probably the other teachers cited, do mean by ‘modelling’. In order to try and do this I will now explore an incident in my own classroom, where I use the term modelling to describe what I was doing.

I had chosen the Fibonacci sequence as the basis for a series of lessons. I wanted the class to feel competent at both recognising and describing number patterns and I wanted them to engage with texts that were available from the wider mathematical community about this particular number pattern. The intention was for pupils to work out the meaning of these texts and to ‘translate’ them into their own words. In each part of this series of lessons I think I used models as a way of helping the pupils in their quest to learn.

The first part of the lesson consisted of writing a Fibonacci sequence on the board.

1, 1, 2, 3, 5, 8, ...

I asked the pupils to work in pairs to figure out how this sequence worked and how they could describe it. Each pair decided on a description and wrote it down. I asked the pairs in turn to read out their description of the number pattern and I wrote these on the board. There was a variety of different wordings, although some pupils produced a similar description to others. Here are some.

“I add 1 is 2, 1 add 2 is 3 and so on to get the sequence.” This, or something very like it, came from five pairs.

“You add the number in front to get the next” from Angie and her colleague.

“You add the two numbers in front to get the next one” from Gemma, her colleague.

The class then considered which description was the best at describing how Fibonacci’s sequence works. There were quite a lot of comments. Pupils felt that the first one was only useful at the very start of the sequence and they knew the sequence carried on a long way. Angie’s definition was good but it might mean that you have to add lots of the numbers and so Gemma’s was best because it told you exactly what to do.

Now I would say that the class were considering models of how one might describe Fibonacci’s sequence. I would say pupils could consider them and learn from them because their colleagues wrote the descriptions. They could say that this one is better than that one, and they could also believe that they could produce the best description. I suppose that you or I could produce a better description of Fibonacci’s sequence: one that is considered better than Gemma’s by our colleagues in the mathematical community. However, I think this would not be in the sphere that these pupils believe they can aspire to and therefore would not be a model they could consider and learn from in the same way.

The quality of the description or way of wording the description that is the model needs to be one that the pupils themselves can produce. The mathematics they read in their textbooks or see written on printed posters or even on the blackboard may well be completely beyond their reach. If that is so then it is not empowering as a model and therefore not helpful to the pupils. Gemma’s description was in a language used by the pupils in the classroom and contained sufficient details expressed in a concise way. It could therefore be seen as a step along the road to mastering the mathematics register. It made sense to the rest of the class and they could see that the details it contained were important and that it was more general than the first type of description given. They recognised that it was a more mathematical way of describing things than using particular numbers and they chose it as the class description. At this time the ‘qualities’ of generalisability and conciseness that are considered important in mathematical communication were not spelled out, but nevertheless I feel that the class took a step along the road to understanding these qualities.

Here then I use the word ‘models’ to portray descriptions of a number sequence produced by the class after some thought. They are good models because

- they are produced by members of the class and were all reasonable attempts at the task;
- there is a variety of approaches to choose between;
- there is no pressure to choose one model over another;
- there is no power issue as there would have been if, for instance, I had put my suggestion in amongst them;
- they express a set of qualities that it is hard to describe and understand.

Using models in this way the pupils did not just learn about Fibonacci’s sequence but could learn about some of the subtleties of writing in the mathematics register. The models provided were well within their grasp and were available to think about without having to use one ‘right’ way. The focus of the lesson was the task of describing well, about coming to an understanding of the best way to do that. Once the models were written on the board...
they were de-personalised and therefore available for scrutiny; the task that everyone was contributing to was improving the description.

The lesson continued using another strategy that again involved models. The task that I asked the pupils to do was to look at some descriptions of where the Fibonacci sequence occurs in nature, that I had downloaded from an Internet site. These were about generations of bees, generations of rabbits, the markings on a pineapple’s skin and the growth of a plant.

The language used in these pieces of writing was not especially difficult and there were some small pictures, but these pupils were not practised at using texts in mathematics at all. Although they had a textbook, we rarely used it and they were very poor at gaining information from it when we did. These pieces of writing about the Fibonacci sequence were texts from outside their experience, from a part of the mathematics community. I used such texts for all sorts of reasons, but primarily to convey the message that these pupils could communicate with the wider mathematics community and that the wider mathematics community had something to say to them.

I felt the need to set up a community of practice as a way to encourage this group to engage with mathematics. I started off with the notion that one of the reasons that the group gave for finding mathematics so difficult was that they could not talk about it in the way that they were able to talk about other things in their experience. They had no investment in the language and practices that they saw going on in a mathematics classroom and many of the ways that they saw mathematics happening effectively excluded them. They either did not have or did not use names that embodied mathematical concepts, perhaps because they were not invited to share in the naming process. In a community of practice there is a shared way of communicating, and a member of that community becomes more skilled as they become less peripheral. Therefore, the practice in which I was attempting to include my pupils was that of communicating mathematics: to name mathematical ideas and to talk about mathematics. This might enable them to be better able to think about mathematics and therefore become better mathematicians. Since this practice is about ways of talking and communicating I tend to use the phrase community of discourse.

If I set up a community of discourse in my classroom that had no overlap with the wider discourse of the mathematics community I would not be helping my pupils or indeed doing my job. So this part of the lesson was about just that: pupils communicated with other members of this community of discourse outside their classroom.

The printouts from the Internet I regarded as models: ways of presenting information that could be considered by the pupils and changed to something that suited their purpose better. The purpose in this case was to present the information clearly in a way that their peers would understand and to learn about this process. But this time the models presented were not ‘good’ models as far as the pupils were concerned because
• they were not written in a language that the pupils could readily engage with;
• they were in a format that the pupils found hard to decode.

However, although acknowledging that pupils would not find it easy, I wanted them to develop skills that would allow them to access information in formats they found difficult, and so see themselves as participators in the wider community of discourse. They took the texts and, working together in small groups, began to dig out the information in them. They translated the language into their own. The bees became daddy bee and granny bee and even great-great-granny bee: it seemed that once the bees were named they could be talked about and the relationships understood. This was true in all the groups: even the plant shoots were given specific names, such as one-year shoot, two-shoot and three-shoot.

So these models were useful in allowing the pupils to develop an idea of what their finished work would look like, which this time was not like the model. The ‘quality’ that the pupils were aspiring to was not present in the model, but the model helped them to decide on this quality. The texts from the Internet had to be torn apart and turned into a different text before they met the criteria for quality that this time were set by the pupils themselves. No-one tried to copy the texts or to use anything other than a similar layout of diagram in the posters they produced. They had an idea of audience which was people like them and they attempted to portray what they had understood in a way that others would understand.

During this write-up stage I again used the idea of modelling. This time I asked all of the groups to stick their partially finished poster to the windows and invited them to look at each other’s work. This has similarities to the models used earlier, but also included sharing good practice, putting your own efforts in a position where they can be looked at critically, and taking time to reflect. No-one had finished, all had had subtly different ideas to share and this type of exercise works best in a co-operative community where we are all looking to produce the best we can, rather than a competitive one where a pupil is trying to be the best in the class.

So from this short episode there are important ideas about modelling:
• There can be ‘good’ and ‘bad’ models but that ‘bad’ ones can be useful.
• Models provide an idea of ‘quality’ in ways of working that are difficult to come to an understanding of.
Models allow pupils to work together towards shared high standards.

Models allow pupils to reflect on difficult concepts and thereby allow them access to some of the nuances of these.

Pupils achieve these aims best if the ideas in the model are within their sphere of aspiration.

A model then is any device that allows the pupils to reflect on either a concept in mathematics or a way of communicating mathematics. It allows them insights into that concept or way of working. It is useful when it is difficult to describe exactly what is wanted or when there may be a variety of responses or some individualism in response. It is also a way of giving all pupils the opportunity to consider, talk about and hopefully understand the task they have been asked to do.

An artefact can be used as a model when used in one way but not when used in another. Although generally, the textbook was not suitable for this class to learn from I later used it in the same way as I had used the texts from the Internet. Having had previous experience, pupils were able to view the textbook as a model and not as an immutable authority. They were beginning to be able to converse with the wider community as represented by the textbook.

Modelling then is not one single way of setting up a learning environment. All manner of artefacts can be used as models: books, work done by pupils last year, role play, posters, on-going work by the pupils and many other things, depending on the learning going on. However, all these things must share these important features if they are to be helpful to the learners.

• A model should be designed to give the learners an idea of how they might do something, rather than one single royal road to follow. It follows then that alternative ways of completing a task may be usefully included in the modelling process.

• A model needs to indicate the quality of work that the learner may aspire to. Using texts produced by the learners themselves may achieve this. Discussing the good points of each other’s work can give learners an idea of quality that they believe they are capable of achieving. Obviously there are other ways of doing this, as in the texts from the Internet, which the pupils translated into something within their own sphere, deciding on their own idea of quality.

• A model has to be available for reflection by the pupils. Therefore it is better if it is visible for a while rather than transient. That is, it is better written down than just said, although a chance remark could be a model if it is well remembered.

• A model needs to be available for criticism. It most certainly does not need its Ts crossed and Is dotted. Something that is finished and perfect has a power that a model does not need. If the print-outs from the Internet had been in full colour and properly paginated I wonder whether pupils would have felt the permission to work on them the way they did. A model must be able to be changed by learners to suit their own purposes.

• A model should be empowering to learners. It must not have a power agenda of its own. Therefore, teachers must take care when producing models themselves. The power or authority of the teacher can make it hard for the pupils to criticise and this removes some of the important features of a model.

• A model must be within the learners’ sphere of aspiration: that is, learners must believe that they can produce the quality shown by the model. This quality is difficult to describe, because a model can show how not to do it as a way of getting the learners to decide on their own ideas of quality. The pupils have to be able to say something about the model (other than “I don’t get it”) and therefore it is a good idea if it is at a level that they can understand. The class providing models for one another goes a long way to fulfil all the above, and also has further potential. Sharing ideas in this way can bring together a set of individuals all intent on completing the task in hand to the best possible standards. The focus of the model is clearly on the task and therefore this can be a tool in creating a learning community. I hope that it also creates a community of discourse, with mathematics to discuss and learn about, names to use and ways to use them when discussing it.

In short a model will be

• available for criticism and have no power agenda;

• not restrictive and in no way purport to show the right way to do something but rather to give ideas on how something might be done;

• inclusive rather than exclusive, a property that is not a feature of some written mathematics;

• flexible and easily changed or developed.

Using models seems to me to be important, because it allows the pupils to develop their own skill and judgement in using and communicating mathematics. Models offer ways of working but not prescriptions. They allow the pupil to take ownership of their mathematics whilst still offering them support while they do this. Offering models in the mathematics classroom is an important idea.

---

Clare Lee is a Research Fellow at King’s College, London and previously taught at Stoke Park School in Coventry.
The attached document has been downloaded or otherwise acquired from the website of the Association of Teachers of Mathematics (ATM) at www.atm.org.uk.

Legitimate uses of this document include printing of one copy for personal use, reasonable duplication for academic and educational purposes. It may not be used for any other purpose in any way that may be deleterious to the work, aims, principles or ends of ATM.

Neither the original electronic or digital version nor this paper version, no matter by whom or in what form it is reproduced, may be re-published, transmitted electronically or digitally, projected or otherwise used outside the above standard copyright permissions. The electronic or digital version may not be uploaded to a website or other server. In addition to the evident watermark the files are digitally watermarked such that they can be found on the Internet wherever they may be posted.

Any copies of this document MUST be accompanied by a copy of this page in its entirety.

If you want to reproduce this document beyond the restricted permissions here, then application MUST be made for EXPRESS permission to copyright@atm.org.uk

The work that went into the research, production and preparation of this document has to be supported somehow.

ATM receives its financing from only two principle sources: membership subscriptions and sales of books, software and other resources.

Membership of the ATM will help you through

- Six issues per year of a professional journal, which focus on the learning and teaching of maths. Ideas for the classroom, personal experiences and shared thoughts about developing learners’ understanding.
- Professional development courses tailored to your needs. Agree the content with us and we do the rest.
- Easter conference, which brings together teachers interested in learning and teaching mathematics, with excellent speakers and workshops and seminars led by experienced facilitators.
- Regular e-newsletters keeping you up to date with developments in the learning and teaching of mathematics.
- Generous discounts on a wide range of publications and software.
- A network of mathematics educators around the United Kingdom to share good practice or ask advice.
- Active campaigning. The ATM campaigns at all levels towards: encouraging increased understanding and enjoyment of mathematics; encouraging increased understanding of how people learn mathematics; encouraging the sharing and evaluation of teaching and learning strategies and practices; promoting the exploration of new ideas and possibilities and initiating and contributing to discussion of and developments in mathematics education at all levels.
- Representation on national bodies helping to formulate policy in mathematics education.
- Software demonstrations by arrangement.

Personal members get the following additional benefits:

- Access to a members only part of the popular ATM website giving you access to sample materials and up to date information.
- Advice on resources, curriculum development and current research relating to mathematics education.
- Optional membership of a working group being inspired by working with other colleagues on a specific project.
- Special rates at the annual conference
- Information about current legislation relating to your job.
- Tax deductible personal subscription, making it even better value

Additional benefits

The ATM is constantly looking to improve the benefits for members. Please visit www.atm.org.uk regularly for new details.

**LINK:** [www.atm.org.uk/join/index.html](http://www.atm.org.uk/join/index.html)