Alec Richardson raises some difficult issues about how we learn and how we assess.

TWO PUPILS’ WRITTEN WORK

During my PGCE at Bristol University I taught a small group of ten year 9 pupils in Cotham Grammar School. Two of the pupils, Andrew and Johnny, worked together as a pair, but there were some interesting contrasts in the ways in which they worked. My teaching aim with this group of ten pupils was to stretch their knowledge and investigation skills in relation to the areas of shapes.

The written work shown here is from the third lesson. The previous two lessons included the following:

- an assessment session (discussion) to establish the pupils’ knowledge so far
- derivation and use of the formula for the area of a circle, using triangles
- finding the areas of multi-sided regular and irregular shapes, using triangles
- a pupil-led exploration of ‘equable shapes’ introduced with the example of a 4 by 4 square.

We began the third lesson (see lesson plan on page 22) by collecting together three equable shapes which had emerged from the previous lesson:

- circle of radius 2 cm – discovered by manipulation of \( \pi r^2 \) and \( 2\pi r \)
- square of side 4 cm – used to introduce the topic of equable shapes
- equilateral triangle of side ‘a tiny bit less than 7 cm’ – discovered by a number of pupils using trial-and-error construction and measurement.

I then suggested that each pupil drew a circle with radius 2 cm and then a square with side 4 cm with coincident centres. Further prompting was given by asking “What about the triangle?” “What about other shapes?” “Is there a way of finding and predicting equable shapes?” and “I’d like a written account of what you find out and why it works.”

From this point Andrew and Johnny worked, scribbled and drew for about twenty minutes. I then heard the following dialogue:

**Andrew:** “OK, how shall we write it up then?”

**Johnny:** “Don’t care, what’s the point .... there’s no-one out there who really wants to know about it.”

Andrew spent the remaining twenty minutes of the lesson producing the piece of written work shown. Johnny decided that his work, also shown, was sufficiently self-explanatory. He then spent the rest of the lesson enthusiastically sharing his findings with some of those who had not yet found a pattern!

From reading Andrew’s work and from listening...
in on Johnny's explanations there is little doubt that both pupils had reached the same conclusions and the same level of understanding. My interpretation of their findings is as follows:

- Any regular shape drawn round a circle with radius 2 cm will be equable.
- The rule applies to a shape with any number of sides.
- In the formula for the area of a triangle, the 2 cm cancels out with the \( \frac{1}{2} \) in the formula to make the area the same as the base.

I also made the following observations:

- No mention at all was made of investigating irregular shapes, but both Andrew and Johnny seemed convinced that their rule applied only to regular ones.
- There was no discussion of how the shapes were visually approaching a 2 cm circle as the number of sides increased – despite Johnny having superimposed a triangle, a square, a pentagon and a hexagon on the same circle.
- “Drawing the shape around the circle” was sufficient description of the process; no mention was made of tangents or of sides just touching the circle.
- Despite having been given the example of the square fitting around the outside of the circle, Johnny and Andrew decided to explore drawing shapes inside the circle first. When questioned they confessed it to be because they found it easier to draw the shapes inside.
- Both pupils were confident enough with their algebra to demonstrate how the triangular area formula was used, but neither was
confident enough to leave this as a proof. They were not convinced without numerical examples, and they preferred to use numerical examples to convince others.

It is significant for me that the majority of their findings and of my observations can be drawn from Andrew's written work. Johnny's written work, however, yields very little if assessed in isolation.

This lesson, and the contrast between Andrew's and Johnny's work, has stuck in my mind for a very personal reason. As a learner through both school and university I have always been highly motivated to understand the ideas presented to me. My motivation quickly runs out beyond this point; I tend to find myself resenting any time spent proving to others that I do understand. The risk involved is that I might not understand as well as I think I do and I might deny myself the chance to be corrected.

In contrast, my attitude towards teaching is vastly different. If someone wishes to draw on my time because they do not fully understand then I do not resent a second of it (within practical limits). In the lesson with Andrew and Johnny I became caught up in the contrast between the motivations of the two boys. I found myself confronted with the same set of emotions which I have always experienced when approaching a coursework deadline. From this lesson, and from the two pieces of written work the boys produced I have arrived at two major conclusions.

As a learner I now understand why I was a constant cause for concern to my teachers (I have always performed well in final examinations and yet have rarely passed a mock exam). I have never responded well to my teachers' methods of assessment, and have made it difficult for them to plan my learning.

As a teacher I need to develop two complementary sets of strategies. The first set will allow assessment of pupils' learning based on all the forms of communication which they choose to employ. The second set will encourage pupils to communicate in a way which allows them to be properly assessed, and which aids self-assessment.

When he wrote this Alec Richardson was a PGCE student at the University of Bristol. He now teaches at Backwell Comprehensive School, North Somerset.
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