The Sound of Logarithms

If you open up an electronic device with a non-digital volume control you will see that such controls are based on a potentiometer (a resistor with a variable connection along its resistive material). A typical potentiometer is shown on the right. You can see that this volume control is marked 2MΩLG. This means “2 Mega ohms logarithmic scale”.

When you turn the spindle the resistance (between the middle connection and either of the outer ones) does not change in a linear way with angle of rotation (i.e. not proportionally). Rather, it changes logarithmically.

What are logarithms doing in your volume control?

When we increase the volume from a sound system, we increase the power output from its loudspeakers (or headphones). Let us say that we double the loudness by increasing the power from $P_1$ to $P_2$. If we then want to increase the loudness from $P_2$ to $P_3$, careful measurements show that the same increase in loudness (e.g. doubling) does not correspond to the same increase in power.

In fact to increase the loudness by the same amount we require that the ratio of the initial and final values of power is the same. In other words if the increase in loudness is to be the same we require that $\frac{P_2}{P_1} = \frac{P_3}{P_2}$.

Thus, if $P_1 = 0.1$ watts and $P_2 = 1$ watt then $\frac{P_2}{P_1} = 10$ then we require that $\frac{P_3}{P_2} = 10$. Putting these two equations together we see that $P_3 = 10$ watts which is a hundred times the magnitude of $P_1$.

Logarithms express this sort of relationship since $\log \left( \frac{P_2}{P_1} \right) = 1$ and $\log \left( \frac{P_3}{P_2} \right) = 1$.

In addition $\log \left( \frac{P_3}{P_1} \right) = 2$. The logs show that the $P_3$ is twice as loud as $P_1$.

The fact that equal increases in loudness require equal ratios of the start and final power shows that the ear responds to sound logarithmically. Many biological responses are of this type showing that logarithms have some very direct applications and were not invented by mathematicians to torture generations of maths students.

Note on log scales. Whereas a linear scale might be typically marked off in equal steps of 1, 2, 3, 4, etc, a logarithmic scale might be typically marked off in equal steps with values $10^1$, $10^2$, $10^3$, $10^4$ etc. Thus equal steps produce equal ratios rather than equal absolute changes.