**Stored Energy**

**Introduction.** We all rely on energy being stored in various ways. This may be medium term storage in batteries, short term storage in capacitors and springs or a whole variety of other storage techniques. It is interesting that the formulas for different types of stored energy have the same form.

**Kinetic Energy**

The kinetic energy (KE) of an object of mass $m$ is given by the work required to accelerate it from rest.

$$ F = ma = m \frac{dv}{dt} = m \frac{dx}{dt} \frac{dv}{dx} = mv \frac{dv}{dx} . $$

From this we have work done (WD)

$$ \int_0^x F dx = m \int_0^v dv = \frac{1}{2} mv^2 $$

Therefore KE = $\frac{1}{2} mv^2$

**Electrical Energy**

The energy stored by a charged capacitor is given by the work needed to transfer charge from one capacitor plate to the other.

The charge held by a capacitor is given by $Q = CV$ where $V = \text{p.d.}$ across the plates and $C = \text{capacitance}.$

The WD in moving $\delta Q$ is $V \delta Q$

Therefore WD in charging is given by

$$ \int_0^Q \frac{Q}{C} dQ = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 $$

Thus energy stored = $\frac{1}{2} CV^2$

**Spring Energy**

The force of a stretched spring is given by $F = kx$, where $k$ is the spring constant and $x$ the extension.

The WD in stretching the spring is given by

$$ \int_0^x F dx = k \int_0^x x dx = \frac{1}{2} kx^2 $$

Therefore the energy stored in the spring is given by $\frac{1}{2} kx^2$

**Rotational Energy**

The rotational equivalent of Newton's second law is

$$ C = I \frac{d \omega}{dt} = I \frac{d \theta}{dt} \frac{d \omega}{d \theta} = I \omega \frac{d \omega}{d \theta} $$

Where $C = \text{the couple}$, $I = \text{moment of inertia}$, $\omega = \text{angular velocity}$.

WD in rotating a body through $\theta$ against a couple $C$ is

$$ \int_0^\theta C d \theta = I \int_0^\omega d \omega = \frac{1}{2} I \omega^2 $$

Thus the energy stored by a rotating object is give by $\frac{1}{2} I \omega^2$