SHAPE AND SPACE AT KEY STAGE 2

Jenni Back, Chris Brooksbank and Geoff Faux discuss the learning resulting from a variety of geometry tasks.

This all started with a letter from Chris to MT197 in which he wrote, ‘My hunch is that working with shapes is particularly useful for visual learners and does help children see patterns ... I wonder if you have any proof supporting my hunch, or alternative suggestions?’

Geoff: Chris’s letter caused Jenni and me to talk about what was important to us in shape work with KS 2 children. We wanted to respond and we invited Chris to join our conversations. Below is a distillation of our emails and face-to-face conversations on a morning when Chris and I worked together with some of his children.

We all live with the numeracy strategy. Working on this article made us look at both the advantages and the constraints of working within the strategy. In a conversation after a working session in school, Chris and I were very conscious that, again this year, there have been some major additions to the unit plans. Many of these are (individually) good, but this continual tinkering from the top is disabling teachers. We swapped stories about introducing a new maths scheme into school and our memory that, the first time through, we had each had felt it necessary to cover all the material. The following year, we knew the scheme and were conscious that we became selective and professional in deciding what we could do another way and what we could just leave out. In our heads was the idea that the authors of the scheme had worked it out really well and if we missed or skipped we might somehow be doing our children a disservice. The unit plans give out the same message and inhibit both of us. We feel a need to work through each one before we can stand back and make our own decisions about where we need to add and what can be ignored. So we asked ourselves what would we want to stress in shape and space in KS 2 and what do we regard as excess packing? To work on such a global question, we decided to work on a number of activities with children in school and then to stand back and share what we thought was important about that particular activity.

Chris: With a very mixed group of Y4 and Y5 children, I started by allowing the children to choose if they wanted to take part in a ‘hexagons task’, which I briefly explained, and then let the children complete for homework. I planned the task so that it would be accessible to children who liked visual/spatial activities. We started by looking at cut hexagons: card hexagons cut into constituent triangles (six in each hexagon), parallelograms (three in each hexagon) and trapeziums (two in each hexagon). As a follow-up, I asked the children to connect lines on a hexagon and look for/show shapes and patterns (figure 1).

The next day I introduced the idea of looking at the whole shape and seeing how it can be split into equal-sized parts to make fractions. We started by looking at the homework, which all the children seemed to have enjoyed. We talked about the fractions work they had been doing in class (the current unit plan) and tried to relate it to the shapes we had designed.

On blank record sheets we completed the $\frac{1}{2}$ and $\frac{1}{4}$ together, and then talked about how else we could divide the shape. Mary (Y4) struggled with drawing the lines accurately and labelling the parts she made. She found seeing equal-sized shapes difficult. Mark (Y5) got quite animated and enjoyed playing with the shape. He soon identified all the easier divisions – $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$ – and after some talk recognised that the thirds (three parallelograms) could be split again into halves, making sixths (figure 2).

Notice on the bottom line how the first attempt at dividing into five equal parallelograms...
works, but then there is a problem about equal areas in the next two figures on that line. Mark came up with an improved dissection into five parts on the next sheet (figure 3, middle of the second row).

There are some interesting dissections here. Some are completely correct. You can see a development in thinking from dividing up into, say, 30 triangles, to combining adjacent triangles to make \(\frac{1}{2}\) in a completely different way from the way \(\frac{1}{5}\) was constructed in figure 2. After this, there are some more experiments with large numbers of triangles to the centre before an attempt to combine them again in the last one.

Ned (Y4) watched the quick work of Mark (Y5) and quickly set about developing similar ideas. The group was working fairly independently at this stage, and at very different paces. I left the group with the challenge to try to find all the fractions from \(\frac{1}{2}\) to \(\frac{1}{15}\) as a further homework.

When we re-met, the results were very mixed. Again the children enjoyed the task. There were some excellent attempts, but some ‘mis-takes’ were also evident – some children labelled shapes of different areas as the same fraction. Area (cutting pies, etc) is commonly used as an entry to work with fractions. The struggle of redrafting and re-drawing that these children got involved in raises the question ‘What other shape and space activity can also feed into children becoming effective at ‘seeing’ area?’

**Geoff:** I am an enthusiast for Cuisenaire rods. I see the ratio of the lengths between the various rods as another way into fractions. I had the chance to work with a Y4 group in a different school. I have been trying to get to grips with the current changes to the strategy that put a greater emphasis on knowing tables and number bonds at Y3-4. As a bridge to this, I wanted to investigate whether it might be helpful to push the idea of seeing multiplication as a packing of rows of cubes of equal length to form slabs which then build a larger cuboid.

As well as a good supply of Cuisenaire rods, I have a box of Cuisenaire prisms and cubes. The prisms are \(\frac{2}{7}\) (red), \(\frac{3}{7}\) (green), \(\frac{4}{7}\) (pink), right up to \(\frac{10}{7}\) (orange) and then a set with \(\frac{3}{7}\) base up to \(\frac{3}{7}\) \(\times\) \(\frac{6}{7}\) and a set with \(\frac{4}{7}\) base right up to \(\frac{4}{7}\) \(\times\) \(\frac{6}{7}\). Each is colour-coded so that there is both a yellow and a green \(\frac{3}{7}\) \(\times\) \(\frac{5}{7}\), implying that this prism can be filled with either yellow (5) rods or green (3) rods. The largest cube in the set is \(\frac{10}{7}\) \(\times\) \(\frac{10}{7}\) \(\times\) \(\frac{10}{7}\). In each size, there is both the cube and sufficient slices to make up a cube. So, for instance, eight \(\frac{7}{7}\) \(\times\) \(\frac{7}{7}\) \(\times\) \(\frac{1}{7}\), two \(\frac{7}{7}\) \(\times\) \(\frac{7}{7}\) \(\times\) \(\frac{2}{7}\) and one \(\frac{7}{7}\) \(\times\) \(\frac{7}{7}\) \(\times\) \(\frac{7}{7}\) (see figure 4).

As an oral/mental starter, I worked with the multiplication chart (Gibbon, 2005) and doubling. We worked with numbers in the twenties and then forties and moved on to multiplying by four by doubling and doubling again. This was, perhaps,
seen by them as unconnected to the shape activity that followed, but I wanted to pick up doubling as a mental skill, later in the lesson. Next, I produced the middle drawer of the prisms and cubes and asked each child to choose a block. We negotiated a bit so that everyone had one that was a different size, not just a different colour.

Holding up a one centimetre cube, I asked how many one centimetre cubes they each thought was in their own block. Below are their guesses.

- Tim 21
- Diana 44
- Brian 62
- Douglas 67

So I asked, “Can you now get a more accurate answer?” There was a lot of ruler measuring of edges and a lot of adding up of small numbers. I managed to keep quiet. After about 8 minutes, I quietly built a $3 \times 3 \times 4$ prism from four $3 \times 3 \times 1$ slices and left it sitting there. Diana noticed first and immediately made up one of the slices from three green rods. She had an answer of 36 for her prism ($3 \times 3 \times 4$) and a certainty that it was right. The others were now into working with slices, and quickly had verifiable answers (figure 5).

To move on, I produced the yellow $5 \times 5 \times 5$ cube and asked how many one centimetre cubes it might have in it. They all went to work. Slices, five yellow rods and adding in fives just seemed to happen. They worked together on adding up the layers: one layer = 25, so two layers = 50 (by doubling). We had worked on doubling 25 at the start of the lesson. So four layers = 100 ($2 \times 50$) and add a final layer of 25 to give 125.

I opened the bottom drawer and produced the orange cube. They all immediately said that it had 1000 centimetre cubes in it. But from our work and talk in the earlier part of the lesson, I was not sure whether this was rote remembered or usable knowledge. I produced some $10 \times 10$ slices, and we talked and counted in hundreds. They also had a certainty that the slice was a hundred. Again, I asked myself what did I know about each child’s connectivity within that knowledge. I moved sideways to ask how many yellow cubes there were in the orange cube. This was quickly sorted out by all four of them to be eight. And I asked, “So what is eight lots of 125?” Brian wrote 125 down eight times in a column and added it up to get 1040. How wonderful! I was able to talk with him about the inefficiency of adding when the numbers are big.

So I took two yellow cubes, put them together and we mentally doubled one hundred and twenty-five to get two hundred and fifty. Now putting two pairs together to get two lots of two hundred and fifty; ie, 500. And the 1000 seemed to come surprisingly but comfortably for them by doubling again.

Was this about shape and space or about multiplication? I have to answer ‘about both’. But also neither. I know that time, area, volume and angle are all difficult measures. Area and volume are often used as paths to multiplication and division. So part of what I was exploring was how much practical area and volume work do we need to do at Y3 and Y4 before we can be sure that an individual child has enough experience to use their internal visualisation in working with multiplication? From this brief hour, I would say (for most) a lot more than I managed on that morning. For me, effective learning must involve the learner with the triad ‘manipulate – get a sense of – articulate’ (Mason et al, 2005). I think I managed to set up the lesson so that they had a smooth experience of

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Figure 4

Figure 5
all three of these learning states. They had
- an accessible starting point (how many unit cubes in this big block?);
- some space to conjecture (all the initial work with rulers);
- a way for them to be able to check, and talk about, the accuracy of their work;
- a complexity of different possibilities and enough real material, not a picture of the material, so that they would be able to reject unproductive things and work on from what they had within their own heads, rather than feeling they needed to ask what was in the teacher’s head.

The articulation of all the space-filling, and putting a measure on that filling and counting in threes or fours, was the end where we managed to double 125 three times to get 1000. Whether any of them extracted the ‘general’ idea that any doubling of the dimensions of a three dimensional shape will always result in multiplying the volume by 8, I very much doubt. But I live with the hope that this experience lies somewhere buried and may be called upon by some of them later, in working with volume (or area).

**Jenni:** Working at the opposite end of the country to Geoff and Chris, I was not able to collaborate with them in working with the same groups of children, but I have also been exploring ideas about challenging Y5 children to approach mathematics as active learners. This has involved working with a class of pupils on the topics prescribed in the unit plans. This term, I have been with them for one of their maths lessons each week.

In one lesson I gave pairs of children a number of shapes: equilateral triangles, quadrilaterals, pentagons, hexagons and circles. I also gave them some statements about shapes, such as ‘have three sides’, ‘are polygons’, ‘have a right angle’, ‘have four sides’, and some names of shapes. I asked them to make statements by joining up the names with the statements and to decide whether they were always, sometimes or never true. Some of the children spent a lot of time sorting their shapes and matching them with their appropriate names – the class was a low-attaining group with quite a few children for whom English is not their first language, so this didn’t surprise me. The others began to build sentences and discuss with their partners whether their statements were ever true. Having already met a number of worksheets in class which encouraged them to identify different shapes and name them, they still found it difficult to consider the properties of these shapes when confronted with a wide variety of different shapes. They also struggled to apply logical rules to their statements.

As Geoff said earlier, the unit plans offer teachers a route for ‘delivering’ the strategy, but the three of us seem to have a sense that in covering the quantity it is too easy to miss opportunities to help children to learn and understand mathematical concepts at a deeper level. We tend to knock geometry out of children at the top end of primary school and on into secondary school and not offer many geometrical representations of results, so that we lose a chance to work with multiple contexts and the opportunity for pupils to make their own connections between algebra and geometry.

In this area between geometry and number there are many NRICH problems that encourage children to think spatially with concrete materials about number and number patterns One example is ‘Up-and-down staircases’ (figure 6).

**UP-AND-DOWN STAIRCASES**

One block is needed to make an up-and-down staircase, with one step up and one step down.

4 blocks make an up-and-down staircase with 2 steps up and 2 steps down.

How many blocks would be needed to build an up-and-down staircase with 5 steps up and 5 steps down?

Explain how you would work out the number of blocks needed to build a staircase with any number of steps.

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This problem elicited solutions from infant school pupils as well as some older children.

Chloe: I worked this out practically, by building a model using Lego® mega blocks. By doing this, I was able to understand that however many steps up or down you require you must therefore have that amount of blocks in the middle, and then you have one fewer
Kaan: I drew this diagram (figure 7).
I concluded that if I multiply the number of steps with itself I can find the number of blocks. For example, if I want to build:
- a 6 step up-and-down staircase, I will need $6 \times 6 = 36$ blocks;
- a 7 step up-and-down staircase, I will need $7 \times 7 = 49$ blocks
- a 8 step up-and-down staircase, I will need $8 \times 8 = 64$ blocks . . .

Rachael: I noticed that if I take one half of the staircase and put it on top of the other, I have a square shape. This shape is 5 bricks across and 5 bricks down. However many steps you have up and down is the same as the number of bricks across and down in the square. So if the number of steps is $n$, then the number of bricks is $n \times n$.

These responses are remarkable in the connections that are being made here between the numbers and the shapes. My experience supports the idea that the geometry helps them to work more effectively with the numbers and facilitates conjecturing and generalising.

Chris: So how does all this activity connect to my initial question? Am I any nearer to having evidence to back my gut feeling that I need to spend more time working practically with shape and getting the children to ask questions? How can I know that the children have benefited from being asked to work on seeing pattern?

I think we have shown the value of setting tasks that suit the learning styles of children, that broaden experience and attempt to link number, shape and space, but most importantly that children learn best when they enjoy their learning in a relaxed, positive and focused atmosphere. Working with Geoff was very enjoyable and really helped me see again the power of open-ended learning and positive reinforcement. We worked together with a Y4-5 group on 9-pin square geo-boards. It gave me some very good evidence of the knowledge the children had of shape. They certainly had a good grasp of shape names, but did seem to find area difficult. They really enjoyed making different shapes using the elastic bands, though their enthusiasm and pleasure did need some teacher direction to help keep the focus and find answers. Sorting the different shapes made into three groups was a particularly good task. What was particularly pleasing was that every single child succeeded at their own level.

In the discussion after the session, we covered a wide range of issues, perhaps the most significant being the need to provide appropriate challenge. None of us are convinced that all the geometry tasks within the strategy are necessarily good for children or aimed at the right age range. The Y6 work on angle, for instance, could, we think, be moved up to KS3.

Using geometry needs good stimulus and interesting structures that are accessible. Perhaps this is where the three of us might go next.

References

Jenni Back works at Middlesex University. Chris Brooksbank is head of Flookburgh Church of England school, South Cumbria. Geoff Faux, retired from maths advising, works as an assistant lecturer with the Open University and, on a voluntary basis, in the primary school his own children attended in times past.

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