Early 30 years ago in MT141, Alan Wigley discussed two models for teaching mathematics. He contrasted the path-smoothing model to the challenging model. Path-smoothing involves the teacher deciding on the type of problem, leading students through a procedure to solve a problem and providing repetitive exercises for the students to practise until they can use the procedure with the minimum of error. Challenging involves setting a problem or context that can be tackled with a variety of approaches, giving students time to explore and conjecture and formalising techniques so that they are consistent with the syllabus and can be applied to other problems. It is our contention that these two models are still prevalent in today’s mathematics classrooms, although now with an updated focus on, respectively, small steps and agency.

In the article, Alan Wigley proposed the challenging model as a way of integrating exploration and instruction and not of rejecting one or the other. However, he warned against an uncritical “cosy consensus” that presented mixing the two as common sense: “The problem is not whether one should use a mix of methods (of that I have no doubt) but precisely how the blend should be achieved”. It was not to be achieved, he continued, by juxtaposing open-ended tasks alongside a didactic approach. Instead, the challenging model required the teacher to “learn to live consciously with the creative tension which exists between exploration and instruction – in deciding, for example, whether to tell or not to tell about a particular matter”.

In our view, the idea of a creative tension continues to be the key factor in mathematics classrooms. Indeed, we would expand the idea from the students’ side. In our classrooms the creative tension involves students expressing their agency by participating in the direction of the lesson. However, proponents of the path-smoothing model in its modern guise continue to dismiss the issue of creative tension entirely. Over the last 30 years more seems to have stayed the same than to have changed. We find this puzzling. Our article attempts to update Alan Wigley’s categories and to suggest reasons why the division is as pronounced as ever.

Path smoothing - small steps

The National Centre for Excellence in the Teaching of Mathematics (NCETM) who lead the DfE sponsored mastery initiative advocate small steps in the teaching of mathematics. It justifies the approach on the basis of variation theory, which is “characterised by a carefully constructed small-step journey through learning” (See https://www.ncetm.org.uk/resources/50819). The teacher devises sequences of practice questions that are “minimally different”, that is, only one aspect is changed at a time while the others remain the same. In a set of equations to solve, for example, the coefficient of one variable is systematically changed. Each equation is very slightly different to the preceding one. Students direct their attention to the difference from which, the NCETM claims, they “will make connections and build deep conceptual knowledge”. The process is termed “intelligent practice” (or procedural variation), as opposed to mechanical repetition, because students derive a general structure from the exercise itself.

The NCETM, and the national network of maths hubs it leads, draw on the practice of Shanghai. Charlie Stripp, the NCETM’s director, relates how he observed lessons given by Chinese teachers who were “deploying short, precise steps with clear logical progression to develop the mathematics” and “purposeful pupil activity on tasks employing procedural variation” (See https://www.ncetm.org.uk/resources//48551). Indeed, it seems that there is always the potential to make the steps smaller. One teacher who attempted to break a learning episode on the concept of perimeter into small steps describes how observers told her that she had covered ground that would have constituted six lessons in Shanghai (See https://www.ncetm.org.uk/resources/49239).

For us, the use of small steps constitutes path smoothing. Alan Wigley in his original article acknowledges that even in the path-smoothing model teachers will consciously offer insightful experiences. However, the use of easily-digestible atomised steps denies students the space to explore their own
questions and observations or to participate in making choices about the direction of learning. This problem originates in the NCETM’s interpretation of variation theory solely in terms of the teacher carrying out the variation. When Anne Watson and John Mason started to apply variation theory to mathematics education, it was very much infused with the idea of student agency: “What pervades and informs our practices in teaching is the image of active, arguing learners engaging with examples and, when possible, constructing their own examples and their own objects” (Watson and Mason, 2005 p. 8). When describing an activity called “taxicab geometry”, for example, they relate how learners started to make up their own examples to test a conjecture, which, “reaffirmed an enquiry-based stance to mathematics as a constructive enterprise” (Watson and Mason, 2006 p. 96).

Jerome Bruner calls for students to be given practice in “leaping” and “plodding”. An exclusive focus on small steps, he claims, consigns students to plodding through their mathematics lessons and deprives them of their “rights as a mind” (Bruner, 1963 p. 531). Indeed, a recent study of the first two years of the implementation of the NCETM’s Shanghai-inspired “teaching for mastery” approach shows that it makes no significant difference to attainment of the implementation of the NCETM’s Shanghai-531). Indeed, a recent study of the first two years

rather, Bruner suggests, students should be allowed to take “great leaps, huge guesses”, or, in the context of mathematics, speculate and conjecture, because it is often by leaping that students become aware of what they know and, we would add, what they do not know.

Challenging - student agency

Throughout our careers, and now as heads of departments, we have sought to develop a challenging model of learning mathematics. Continual experimentation with the creative tension between instruction and exploration has led us to a realisation that the concept of agency is central to building a challenging model. In particular, we want to develop students’ agency to enable them to make sense of the contexts and problems we bring to the classroom. We define agency in the tradition of cultural-historical activity theory. Agency refers to “the actual ways situated persons wilfully master their own life” (Van Oers, 2015 p. 19) and involves intentional actions that are aimed at transforming an activity by “explicating and envisioning new possibilities” (Engeström et al., 2014 p. 124). This is our aim in lessons: to develop students who are capable of deciding how they will act in learning mathematics. They regulate their activity, through the structure and guidance we provide, in ways that are consistent with the cultural norms established by the mathematics community. Helen’s model uses learning journeys to give students in mixed attainment classes responsibility for determining their starting points and selecting their tasks; Andrew’s Inquiry Maths model involves regulatory cards that support students in directing their own inquiries into a mathematical stimulus.

Mixed attainment classes and learning journeys

In Helen’s mixed attainment classroom creative tension occurs when teachers and students collaborate to assess current levels of learning and to choose appropriate tasks and approaches. There are two factors in enabling students to do this successfully: differentiation and awareness. Firstly, the tasks must be differentiated in line with the students’ different starting points. As students in a mixed attainment classroom have a range of prior knowledge, differentiation is essential to ensure that all are given the opportunity to build upon their existing knowledge. Differentiation can involve individuals working on different tasks that provide a graduated challenge; or, alternatively, it can involve all students working on the same task that provides opportunities for different levels of reasoning.

Secondly, students need be aware of what they already know and what they need to know next. They can develop an awareness of their current and proximal levels of understanding through the use of learning journeys (see figure 1), which describe the different mathematical skills that are covered within each unit of learning. At a practical level, students complete an assessment task at the start of a unit to identify the skills they have already consolidated. They refer to their learning journey in each lesson to inform their choice of a task and also to reflect on the new knowledge they have gained. Another important way in which students become aware of their own prior learning in mixed attainment classes is through listening to and reflecting on their peers’ contributions during class discussions. The teacher plans and leads the class discussion to ensure the knowledge and methods necessary to enable all students to
start on their chosen tasks are shared. The end-of-unit assessment is used to reflect on progress and the suitability of the tasks each student has chosen during the unit.

![Figure 1: Learning journey. A learning journey for sequences.](image)

Although learning journeys and class discussions can help guide pupils towards making the right choices, it is important that teachers also take responsibility for monitoring students’ task choices and re-directing them when necessary. This is not to say that teachers should have a pre-determined view of an appropriate task. Such an approach risks limiting students’ progress and reinforcing an individual’s negative belief about their ability to learn mathematics. Task selection is a skill that students will develop over time. Even if some students need more support with this than others, they can all benefit from becoming agents of their own learning.

**Inquiry Maths classrooms and regulatory cards**

In an inquiry classroom, the decision about the line or lines of inquiry to follow is an expression of the creative tension between the structure put in place or guidance offered by the teacher and the level of students’ independence and initiative. These decisions emerge from the questions and observations that students make when they are first exposed to the stimulus (known as a prompt in the Inquiry Maths classroom) and from their choice of a regulatory card (see figure 2).

An Inquiry Maths lesson starts with students posing questions about the prompt or noticing its properties. In this phase of the inquiry, students make connections between their existing knowledge and the prompt’s underlying mathematical structure. The teacher uses the students’ responses, which might range from basic questions about definitions to attempts at sophisticated conjectures, as information that informs the nature of the inquiry. If, for example, students do not know the meaning of key vocabulary or a method to carry out a calculation, the teacher might decide to structure the start of the inquiry by telling or explaining to the class. If the majority of students understand a conjecture, the teacher might take a guided approach by suggesting a period of exploration that involves verifying or refining the conjecture. The teacher does not take a decision on the direction of the inquiry alone. Students are offered regulatory cards from which to choose a course of action. A class of 11- and 12-year old students might be offered six cards, while a class experienced in inquiry will be building up to a set of 20 cards.

![Figure 2: Regulatory cards. Six cards (left) for classes new to inquiry; 20 cards (right) for experienced inquirers.](image)

The point of the cards is to promote the students’ agency. Through their selection and discussions about their peers’ selections, students learn how to conduct an inquiry within the discipline of mathematics. Moreover, our evidence shows that they also start to combine cards spontaneously: at first, two cards, one focussed on what action to take and the other focussed on how to carry out the action; then different cards that combine actions; and, finally, a sequence of cards that explicitly maps out steps within the inquiry. Ultimately, students regulate (plan, monitor and evaluate) their inquiry without the support of the cards. At this point, they “wilfully master their own life” within the mathematics classroom and in line with the practices of the mathematics community.

**Two models**

That the path-smoothing and challenging models have endured as separate approaches, albeit in different forms, suggests that there are deep underlying factors that sustain them. We wonder, in particular, how the path-smoothing model has survived if we take into account its limited and even negative results. Alan Wigley gives two suggestions. He points to the predictable nature of public examinations, which make them amenable to the path-smoothing approach. The problem-solving and reasoning demands of new GCSE and A-level exams make this argument less convincing today. Then, according to Wigley, there is the pressure from parents and the public for mathematics to be taught in a way they
recognise. While this pressure undoubtedly exists, it is our experience as leaders of maths departments that parents can be convinced through reasoned arguments (about, for example, mixed attainment groupings) that place the interests of children above any perceived “natural” way of teaching.

We would highlight two other factors that support the two models. The first relates to the definition of mathematics. Our view follows Polya’s that: “Mathematics presented with rigour is a systematic deductive science but mathematics in the making is an experimental inductive science” (Polya, 1973 p. 117). The experimental side of mathematics involves questioning, exploring and conjecturing; the deductive side involves generalising, structural analysis and proof. Mathematics is built on small steps, but, by interweaving induction and deduction, it also relies on great leaps of imagination. The definition of mathematics underpinning the path-smoothing model, at least insofar as the student is concerned, is a logical chain of small steps. It incorporates a very limited form of generalisation from a narrow set of teacher-devised questions. If students follow the path-smoothing model exclusively, they will not know how mathematicians reason or, more importantly, why they reason in the way they do. Once the teacher’s small steps are withdrawn, the danger is that mathematics will cease to have any independent meaning over which students feel they can exercise control.

The second factor comes directly from our experience of working in comprehensive schools over the last 10 years. Austerity, and the ensuing cuts to school budgets, has led to increased class sizes, a reduction in resources and a shortfall of qualified teachers. In these circumstances the path-smoothing model is easier to implement. It is simpler with large classes and fewer resources for the teacher to direct the lesson and students to follow than for the teacher to embrace a creative tension in which teaching and learning are negotiated. It is no coincidence, in our view, that the DfE finances the NCETM’s small steps model at the same time as it cuts funding to schools.

Preservation

In this article, we have suggested that Alan Wigley’s categorisation of teaching models has relevance for classrooms 30 years later. We have updated his definitions, characterising the NCETM’s small steps model as path-smoothing and our own efforts at constructing classrooms based on the creative tension between exploration and instruction as challenging. That these categories resonate three decades later, and in the case of the path-smoothing model, even in the face of evidence of its ineffectiveness, suggests there are deep underlying reasons for their continued existence. We propose that they are built upon different definitions of mathematics and, in the last decade, have been directly influenced by the government’s austerity programme. Alan Wigley ends his article by contending that the benefits of the challenging model can be “better learning and more positive attitudes towards the subject”. Our experiences support this view and our careers, in a way, respond to his request that teachers develop and illustrate the model with their own examples. In the face of the government’s mastery juggernaut, we, in our turn, would encourage teachers to preserve manifestations of the challenging model.

Andrew Blair is Head of Mathematics at Haverstock School, Camden, London. For more information, visit www.inquirymaths.org

Helen Hindle is Head of Mathematics at Park View School, Haringey, London. For more information, visit www.mixedattainmentmaths.com

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