Mathematics education in a time of crisis

Alf Coles continues the discussions in the previous issue of MT exploring mathematics and the living world.

Time for you and time for me,
And time yet for a hundred indecisions,
And for a hundred visions and revisions,
Before the taking of a toast and tea.
from T.S. Eliot's The love song of J. Alfred Prufrock.

The title of this article is not intended to suggest that mathematics teaching is in crisis, but rather that we are engaging in the project of mathematics education in a time of myriad crises in the world, when there is surely no longer time for indecisions and revisions. A question that is being asked by a small but growing number of mathematics educators is whether, or perhaps when, the global context within which we work will make a significant difference to what we do in our classrooms, this was a theme of the last issue of MT.

In this article, I aim to continue the thread of conversations, and offer some ideas arising from the work of a group of teachers and University staff who contributed to a special issue of the Philosophy of Mathematics Education journal (POME) under the theme of Mathematics and the living world (Ernest, 2017). The title of that collection is taken from a suggestion by George Monbiot (2017) to find more emotive words with which to describe concerns with sustainability or the environment.

In that special issue, Mark Boylan (2018) invited us to consider how mathematics classrooms could become sites for students to develop “ecological selves” and for students to come to know mathematics in a “relational” manner, seeing themselves as actors and active in the process of learning, compared to, say, experiencing mathematics as a fixed body of knowledge to be assimilated in a passive manner. Boylan proposed five aspects that could support the development of ecological selves in a mathematics classroom, which are: enchantment; embodiment; emotionality; ensemble; expansiveness (ibid, p.9). I will take the five aspects in turn and explore them briefly, before offering a classroom example to raise further questions. There seem to me no easy answers as to how mathematics classrooms might be sites of engagement with global and societal challenges, yet perhaps no more pressing question.

Enchantment

Linking mathematics teaching to the living world does not have to mean working with real-world contexts. As Boylan (2018) suggests:

Experiencing astonishment, wonder and enchantment in mathematics practises the capacity to experience these emotions in other relationships.
Moreover, mathematics analysis can be a means to understand the animate and non-animate world as wonderful. (p.10)

Another word I find helpful here, alongside astonishment, wonder and enchantment, is “awe”. How might mathematics teaching help engender in students a sense of awe and wonder? There are no shortage of surprising results in mathematics. But simply showing a result that is surprising to us as teachers, or mathematicians, may occasion bewilderment as much as astonishment. There is a need, perhaps, for some preparation, or to generate some sense of expectation, before something is surprising. Surprise can be generated by something being “different” when we expected a sameness, or being “the same” when we expected a difference. For example, the two different open cylinders you can make by rolling an A4 sheet of paper might raise an initial expectation of having the same volume.

Embodiment

Again, without necessarily needing to consider real world contexts, Boylan invites us to consider ways that the body might be mobilised in the classroom.

Embracing embodiment as a pedagogical principle […] suggests the need for enlivening classrooms as places of movement […]. Thus, gesture and the use of materialities and physical representations would be positively encouraged. (p.12)

I am reminded here both of projects such as People Maths (see, for example, http://www.transum.org/software/Fun_Maths/People_Maths.asp) and also Peter Liljedahl’s work on building thinking classrooms, in which a key element is getting students into small groups, working on vertical, non-permanent surfaces. The detail of this project can be found at http://peterliljedahl.com/wp-content/uploads/Building-Thinking-Classrooms-Feb-14-20151.pdf. There is a lot of research taking place on the significance of gesture in learning mathematics and how we might
see mathematical concepts themselves as material and active, changing with each encounter. There are ritualisations that could allow an embodied doing of mathematics, without calling on conscious deliberation, and that can be deconstructed afterwards. For instance, by chanting responses to a teacher tapping out patterns on the Gattegno tens chart.

**Emotionality**

The orthodoxy for much of the last century has been to separate abstract thought from emotion, or affect, and to imagine that, in the mathematics classroom, we are only dealing with children’s intellectual lives. Of course, emotion cannot be separated from more intellectual activities in this way (see Damasio, 1996, for example).

Fostering enchantment and celebrating embodiment in mathematics classrooms entails an acceptance of participants in mathematics education practices as emotional and affective human beings. (Boylan, 2018, p.13).

As with the idea of enchantment, there is a role here for tasks that generate surprise and wonder. Work taking place on mindsets in the classroom could be seen to be an acknowledgment of the importance of emotionality, as are efforts to work explicitly with students on becoming more resilient learners.

**Ensemble**

Dick Tahta (1989) challenged us to consider ways in which mathematics could become a communal activity.

Fostering relationships of respect and reciprocity with other-than-human beings requires fostering relationships of respect and reciprocity with human beings. (Boylan, 2018, p.14).

There has of course been a lot written about group work and programmes such as “complex instruction” in which assessment becomes collective. Perhaps less attention has been given to the kinds of qualities of group work that Boylan is pointing to in the quotation above. As with the earlier notions, there is nothing actually about the content here. We might work on fostering relationships of respect and reciprocity in learning times-tables just as much as learning about climate change.

**Expansiveness**

The motto of a primary school near where I live is, “the unhurried school”. This struck me recently as perhaps particularly appropriate for our current time. Boylan (2018) writes:

Regulated time in education echoes timescales of industry […] with time commodified and separated from the educational focus […]. More expansive timescales involve a slower relationship to learning mathematics […] that emphasise the ‘whileness’ of being with mathematics and mathematical activity long enough for both of these to become worthwhile (p.16).

As a teacher, I liked to find tasks that could extend over several weeks of lessons (one example of which is below). Although sometimes there would be a lull after a few lessons, when students felt like they wanted to move on to something new, I consistently had the experience of benefits from staying with the task, particularly for those students for whom it perhaps took time to gain familiarity with a new context. Interestingly, the current focus on mastery teaching in England seems to be allowing a number of schools to organise their curriculum into longer blocks of time on a topic than they might have done in the past.

**A mathematics classroom**

What might the five aspects above look like in a mathematics classroom? One potentially expansive task, that I liked to use with students when they began the school year, is called 1089 (familiar to many readers, no doubt). The next section is a description of that task and some students’ responses, taken from notes made at the time. I have tried to link each extract to Boylan’s five “e”s. This task has no content connection to ecology but perhaps offers opportunities to develop ecological selves, as defined above.

**The 1089 task**

This is a version of a write-up from Coles (2015). I began by articulating the purpose of the first year of secondary school mathematics for the students as being about “becoming a mathematician” and “thinking mathematically”. I described this as thinking for yourself, and so not asking the teacher if things are right and noticing what you are doing, for example: spotting patterns and then asking why patterns work; writing down everything you notice; being organised and doing things in your head. What follows are three extracts from the lesson.

**Lesson extract 1**

I issued the following instructions, at the same time going through an example on the board:

<table>
<thead>
<tr>
<th>Pick any three-digit number with 1st digit bigger than 3rd</th>
<th>7 5 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reverse the number and subtract</td>
<td>-2 5 7</td>
</tr>
<tr>
<td>Reverse the answer and add</td>
<td>+5 9 4</td>
</tr>
<tr>
<td></td>
<td>1 0 8 9</td>
</tr>
</tbody>
</table>

Figure 1: The 1089 process.

Several comments were made by students that they also got 1089 and the challenge I gave to the class was, “Can you find a number that does not end up as 1089?” There are several reasons behind the choice of this
activity as the first one with our 11-12-year-old students. It is self-generative, that is, having set up the task, I do not need to direct students as to what numbers to try out. They can generate their own examples to try. Also, some students typically become convinced very soon that all answers will end up at 1089. So if a student gets a different answer they can check their working with someone who thinks that is impossible. Thus, the task becomes self-checking for the students. Both these elements leave my attention free to notice aspects of mathematical thinking to highlight to the group. Being able to choose a line of inquiry, within the constrained space of the task, also gives the students an immediate ability to choose a line of inquiry, within the constrained space of the task, also gives the students an immediate experience of having to think for themselves, which was offered to them as being part of becoming a mathematician.

There can be an enchantment to this task beginning, in the surprise that almost everyone in the room has got the answer, 1089, from the initial set of instructions. It is unlikely, actually, that everyone will get the same and a first thing I would do would be to work, with the whole class, to consider any starting numbers that did not come to 1089. Depending on how you interpret the algorithm, answers of 198 are possible and the fact there is not just one answer was important to me in terms of getting away from the idea that mathematics is always black and white. Students can choose different rules for how the algorithm works and end up with different results and, so long as they are consistent, they will be able to progress with the task.

Lesson extract 2

A key practice I take from the work of Laurinda Brown (see Brown, 2004), is a conviction that offering comments about ways of working, “meta-comments”, is a powerful mechanism for establishing classroom norms and routines. Below are two examples of what these looked like in a first lesson on 1089.

This group noticed something about their answers. It proved not to be 100% correct but it’s an example of what it means to think as a mathematician.

This group had an idea which they wrote down and tested and found it didn’t work so they changed their idea, that’s a great example of what it is to think mathematically.

Commenting on activity in this way is part of an attempt to set up a classroom culture in which students’ ideas and questions are valued and in which there is an acceptance that it is okay to make mistakes, from which everyone can learn.

In the second lesson, having worked on the 3-digit problem, some students were ready to move on to looking at the same problem using a different number of digits. A few students were struggling with 3-digit subtractions and I invited them to look at 2-digits for a while. Initially I constrained the other students’ exploration to 4 digits, where there are several different answers. I set up two common-boards (see figure 2) for students to write up their answers.

| 9999 | 10890 | 10989 |
| 9271 | 1520  | 1000  |
| 7164 | 9741  | 4551  |
|       | 8304  |       |

Figure 2: A 4-digit common board.

On the first board, students wrote the numbers they started with, underneath the total it came to (for example, starting with 9271, you end up with 9999, according to figure 2), they wrote their initials next to their number. If another student checked their number and agreed, they came and wrote a tick next to it. When a number had two ticks it was rubbed off, by the original student, and written on the second board of checked results. The challenge with four digits is to predict what total your number will come to. These common-boards develop the self-checking nature of the activity. The checked results board allows every student access to working on the higher-level questions of looking for patterns in the numbers that end up in the different columns.

In the 4-digit case, there are many opportunities for students to begin noticing patterns and making predictions. For example, any number with the middle digits the same will come to 10989. These conjectures were written up on a board with the student’s name attached to them, and tested by others. I chose to begin one lesson getting the whole class to focus on one conjecture before then allowing space to continue their own lines of inquiry.

The common-boards for students to write up their results always feel significant. Initially we might collect all the starting numbers that did not come to 1089, and then there would be boards, as described above, for 4-digits, 5-digits and for students’ conjectures. The embodiment here is limited to students being allowed out of their seats to write on boards, but more significant was the sense of ensemble. Students were working together, checking each others’ answers and working on each others’ ideas by following up patterns noticed by others.

Lesson Extract 3: Algebraic proof

The following sequence took place in a sixth lesson on this task, during which time different students had worked on different questions and different numbers of digits. With twenty minutes to go, I stopped everyone and went through the work shown here, which I introduced as a way of proving what we found out.
for three-digit numbers. Alongside the algebra, I did a numerical example and at each stage of both the numerical and algebraic example I elicited answers for what to write from the class (see figure 3). I then wiped the proof off the board and set the class the challenge of reproducing it and then extending it to prove things they had found out about the problem with different numbers of digits.

\[
\begin{array}{ccc}
  a^{-1} & b^{10} & c^{10} \\
  - & c & b \\
  a-1-c & 9 & c+10-a \\
  + c+10-a & 9 & a-1-c \\
  1 & 0 & 8 \\
  1 & & 9 \\
\end{array}
\]

Figure 3: An algebraic proof of 1089.

I was always keen that students’ first experience of algebra in secondary school was one that was meaningful. I did not expect everyone in these mixed attainment groups to follow the proof or be able to reproduce it but at least they see that algebra can answer the question, “Why is it always 1089?” that they have been asking and which they had not been able answer in any other way.

There is a subtle decision to be made as to when to offer the algebraic proof. Some students can certainly get frustrated wanting to know why the answers always come out in predictable patterns. The task can be structured, therefore, to work with students’ emotionality, occasioning a sense of the need for a proof. Although the algebra is complex, because students have spent so long on the task and perhaps performed that algorithm 50 or 100 times, linked to the expansiveness of time spent on it, I found consistently that many were able to recreate it and then adapt it to prove results for other numbers of digits.

Discussion

In reflecting on the wider themes with which this article began, the sense of global and societal challenges or crises facing the world, as noted earlier, nothing above touches on the content of these challenges. I do believe such content, for example, raising awareness of the mathematics behind climate science, is vital and necessary work to be engaging in and sharing. A mathematics curriculum in which climate change or future cities appear as the topic for a term’s work is perhaps some way away, but no less important. What perhaps can be done now is the development of some of the ways of working pointed to by Mark Boylan in the ideas of enchantment, embodiment, emotionality, ensemble and expansiveness. I can foster some of these qualities with any curriculum and any topic, right now, if I take the care to consider how.

At the end of working on the 1089 task, with the class that was the focus above, I asked the students to write anything they could under the heading “What have I learnt?” since arriving at secondary school. What bits of mathematics they had learnt and also what they had learnt about what it is to think mathematically. One student wrote:

I’ve learnt that you have to think about the problem and not just do the sum. Also you have to maybe carry on thinking about the problem and see if it carries on. You could also have suggestions on why there are problems and how the problem works.

The student continued (abridged) that “We don’t just do the sum, we think about the problem of the sum”. If we are to work on wider issues in the mathematics classroom it seems to me students may need an awareness that mathematics is not just about “doing the sum” and that there are choices mathematicians make, that can be hidden, about the rules and assumptions under which they will operate. Feeling, as a student or teacher, that I can make a difference is perhaps a broad orientation towards the world. Mathematics currently holds such a privileged position in the curriculum that it seems to me likely that students’ feelings of activity or passivity in relation to mathematics can have quite some bearing, over the 10 or more years they are in mathematics classrooms, on their sense of activity or passivity more generally. Or, to quote again from the poem at the start of this article:

Do I dare
Disturb the universe?

References


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