Short tasks in the mathematics classroom: An approach to problem-solving and discussion?

Nicholas Worsley shares the findings of his Master’s dissertation on using short tasks to support problem-solving and discussion in his classroom.

My own teacher training emphasised the value of mathematics lessons that offered pupils the opportunity to think for themselves, to solve problems collaboratively and to discuss and share ideas. As a trainee, I was convinced of the need to provide students with investigative, challenging and creative tasks, which furthered their mathematical understanding. Yet I found that the reality of introducing such opportunities was not always straightforward. In my context, as a secondary mathematics teacher in a South Bristol Academy, I had frequently observed negative attitudes towards mathematics. Students regularly defined themselves as, “bad at maths”. For many students there was a pervasive belief that success in mathematics is determined only by the number of correct answers. Anything other than a page full of ticks constituted failure.

My attempts to introduce rich activities and problem-solving tasks were often unsuccessful. Students frequently interpreted extended problem-solving tasks as an opportunity to chat and many demonstrated low resilience or struggled to access such tasks beyond a superficial level. My initial response was to deliver lessons that were increasingly ‘teacher-led’, since this greatly simplified the process of managing behaviour and increasing student output. Yet I was highly aware of the shortcomings of such an approach. Students were not being presented with sufficient opportunities to think critically, to solve problems or to discuss their ideas. I needed to find a strategy which offered students opportunities for discussion and evaluation, but which could be manageably incorporated within the existing routines of the lesson and was accessible to all students. It was in this context that I developed the notion of the short task.

The short task

The short task was designed as an episode in the lesson, lasting between five and ten minutes, in which students were presented with a prompt question for consideration and discussion that would stimulate their critical thinking and encourage reasoned discussion. Importantly, given my desire to move students away from understanding mathematics as rooted in a right/wrong dichotomy, there was no correct answer to the prompt question offered. Instead, I sought to pose questions which could provoke a range of responses from students and offered them the opportunity to think creatively and critically based on their existing knowledge. There were three groups of prompt question used in the tasks:

1) What are the similarities and differences between the two shapes, objects or questions shown?

2) Which of the following is the odd one out? Why?

3) Which of these questions is the easiest? Which of these questions is the most difficult? Why?

The questions were chosen as they were accessible to all students. I employed the same structure when using these tasks, reading the task aloud, then giving students some time to consider the question, first independently and then with their partner. We then discussed the students’ responses as a class, sometimes recording these on the whiteboard.

To assess the impact of these tasks on problem solving and discussion, I used them regularly with a range of different groups over the course of several months, audio recording and then transcribing the discussions that emerged. Coding and analysing these audio recordings provided a range of insights into the potentially significant value of such tasks. I will share two insights from this research below. Firstly, I will suggest that these tasks function as an effective means of prompting students to consider more deeply the processes and challenges that are required by particular questions. Secondly, I will show the ways in which these tasks can play an important role in fostering student discussion and in developing an understanding amongst students of the importance of formal and precise mathematical language.

Mathematical processes

The first significant observation was the way in which these short tasks proved an extremely effective tool in forcing students to consider deeply the nature of
the mathematical processes they were engaging in.

Put these equations in order from easiest to hardest

\[
\begin{align*}
a) & \quad \frac{x}{4} = 1.5 \\
b) & \quad \frac{2x}{3} = 4 \\
c) & \quad \frac{x + 2}{3} = 4 \\
d) & \quad \frac{x}{x - 4} = 2 \\
\end{align*}
\]

Figure 1: The ranking task.

The ranking task (see Figure 1) exemplifies the ways in which asking students to order questions based on difficulty served as an extremely rich prompt for a Year 10 age mixed attainment group to share their understanding of the challenges presented by different types of equation. Instead of being asked to practise a set of questions independently, students were required to order the equations from easiest to most difficult. Student comments during the discussion suggest that this process of considering the relative difficulty of equations, and of determining which was most difficult, forced them to consider the specifics of how they would solve each equation, and which elements of this process were potentially challenging. Comments included: “E and F are easier because they don’t require you to multiply by the numerator”; “B and C are equally difficult because they both require two steps”; “A is harder than B because you have to multiply a decimal number, but in B you just have to times by 3 and divide by 2.” In being asked to rank questions in order of difficulty, students were forced to consider more deeply the processes that were required to answer each question, and, crucially, to consider these processes in relation to other possible questions. Rather than simply being asked to answer questions individually, students were collaboratively evaluating what specifically made an equation challenging, considering and articulating to one another the justifications for their decisions.

Similarities and Differences?

\[
\begin{align*}
\text{Increase £40 by 10%} & \quad \text{Decrease £46 by 10%} \\
\end{align*}
\]

Figure 2: Similarities and differences.

A similar consideration and articulation of process was observed in a similarities and differences task with a low-attaining Year 8 group on percentage increase and decrease (see Figure 2), two skills often confused by my lower-attaining students. This was a group in which whole class discussion had proved challenging, yet the accessibility of the task meant students felt able to contribute, and prompted several interesting contributions. One student identified the significant difference that, “For one of them you have to add, and for the other you have to take away”, another recognising the similarity that, “For both of them you have to start by finding 10%.” Particularly for lower attaining students, who often struggle with question identification, this process of explicitly forcing them to consider the specific demands of similar questions seems to be hugely valuable. In divorcing the process of noticing differences from the actual answering of the questions, students were offered greater freedom to consider the nature of the demands of each question, and to analyse the question critically. Students noticed that, “One of them will have a decimal in it”, identifying a potential stumbling block and demonstrating considerable awareness of the potentially differing demands posed by two ostensibly similar questions. Student retention of the difference between percentage increase and decrease appeared much improved as a result of the discussion, as did their ability to apply these skills in different contexts.

Which is the most difficult question?

\[
\begin{align*}
3(a - 2) & \quad a^2 + 2 & \quad a + 2 \\
\text{When } a = 10, \text{ find the value of:} & \quad \text{When } a = 10, \text{ find the value of:} \\
2a & \quad \frac{a}{2} & \quad a - 19 \\
\end{align*}
\]

Figure 3: Substitution.

Indeed, student responses suggested that such prompts could even foster a degree of mathematical empathy amongst students, encouraging them to consider how others might approach a question. When identifying the most difficult question from a group of substitution questions (see Figure 3), a consideration of others was central to one student’s justification of why “a²” was the most difficult to evaluate, arguing that, “People might mistake it for a plus two.” Another student identified “a – 19” as the most difficult, stating that, “It’s just a bit harder
to tell because you’re not sure if you should have it as -9 or 9.” The student here identified the difference between evaluating the statement as 10 – 19 and 19 – 10, and in so doing, it seems that he showed an understanding of the crucial importance of the order of numbers in operations involving negatives and was aware that other students might not immediately grasp this nuance. Both responses suggest a depth of thinking about the questions posed, which reveals much more about the nature of student understanding than could be achieved by simply asking the students to complete the six substitution questions independently. In being asked to think about the relative difficulty of questions, students were placed in a position where they felt the need to consider not only what they found difficult, but also to consider what the essential characteristics were of a difficult question. On both occasions, rather than think about this from their own point of view, the students chose to exemplify the difficulty by suggesting what they thought other students might find challenging. The students were actively considering, unprompted, the potential misconceptions that might arise from a given question, and evaluating the relative likelihoods of each. Students were demonstrating an awareness of, and a confidence with, the fact that the risk of such misconceptions permeates mathematics, and were drawing on this awareness to make their decisions.

Mathematical language

As well as yielding interesting insights into the ways in which students considered particular mathematical processes, the short tasks also served as a valuable prompt for discussion and debate, and suggest significant implications regarding the ways in which teachers can encourage students to use mathematical language. When a year 9 class were asked for similarities between two triangles (see Figure 4), one student offered the response that, “They both have the same algebraic numbers”, which I interpreted as referring to the fact that the coefficients of x were the same for both triangles. The task had been deliberately designed to draw student attention to this fact. Yet although I had expected students to notice this difference, the response was nevertheless interesting. Such a statement highlights the ways in which students can be resourceful in drawing on their existing understanding to create the meaning they wish to express. In this context, ‘algebraic number’ functioned just as effectively as ‘coefficient of x’ or ‘x term’, conveying the similarities that the student had observed between the two triangles. When asked for differences between the two triangles, another student identified that, “One’s got add four, add eleven, I don’t know what that’s called”. Here, the student was actively considering the ways in which she was transmitting meaning, aware that she had identified a difference which she was unable to fully express. In saying, “I don’t know what that’s called”, the student seemed to display an awareness of the need to draw on an existing mathematical discourse, and a willingness to do so. Yet although she did not yet have the vocabulary to describe the difference between “3x” and “3x + 11”, her meaning was clear. On both occasions, the task generated an opportunity for me, as the teacher, to provide the vocabulary to allow students to articulate their meaning, introducing the key words ‘term’ and ‘coefficient’ so as to support students to explain themselves. Yet crucially the observation of the difference, and the necessity of the technical vocabulary required to articulate this, came from the students themselves. This seems highly significant. Had the lesson begun with an explanation of what was meant by a term, before asking students to identify the similarities and differences between the triangles, both the ownership over, and the value of, the vocabulary would have been lost. Furthermore, the task was designed in such a way as to force students to engage with these keywords to explain the differences they observed. Rather than the keywords serving merely as an addendum to the lesson, introduced as a formality, they were rendered indispensable as a result of the requirements of the task. The fact that there were several possible differences to be observed meant that students were able to take ownership of this mathematical vocabulary to describe their own observations and thoughts.

Pimm (1991) argued that a major challenge for mathematics teachers is that of, how to encourage movement in their learners from the predominantly informal spoken language
in which they are fluent, to the formal language that is frequently perceived to be the landmark of mathematical activity (p. 21).

The evidence above suggests several potential opportunities. On both occasions, students were able to successfully convey meaning without use of specific mathematical vocabulary, yet at the same time they were beginning to seek this more complex mathematical language. It is possible to see the value of these tasks as functioning at the nexus between informal and formal mathematical discourse, offering both students and teachers a site of mediation through which the language of mathematics can be introduced and understood. In allowing students the opportunity to observe and define differences for themselves, the necessity of the precision of a formal mathematical discourse becomes self-evident. Students can see for themselves the ways in which it is valuable to use specific language to convey meaning. Rather than merely encouraging students to talk about mathematics in the hope that they will eventually move towards a formalised discourse organically, it seems such tasks, if carefully constructed, have the potential to allow teachers valuable opportunities to guide and support students in this process.

**Some possible benefits?**

The regularity with which such tasks provoked interesting and illuminating student discussion and debate suggests that they offer significant potential value to teachers. The examples above highlight the ways in which the careful consideration of tasks can shift the focus away from whether answers are ‘right’ or ‘wrong’, and provide opportunities for rich discussion and debate which are accessible to all students and which offer valuable insights and opportunities. In separating the processes of observation and doing, and moving away from activities centred on a ‘right’ answer, these tasks offered students the opportunity to think about and discuss mathematics collaboratively. While not a substitute for independent problem-solving or investigation, tasks such as these demonstrate that it is possible to provide learners with opportunities for reasoning, discussion and argument without necessarily requiring them to engage in rich or extended problem-solving activities. Instead, careful consideration of the way that questions are posed offers teachers and students the potential for rich and productive mathematical discussion.

Furthermore, the short task was designed as a manageable and self-contained episode within a lesson. I do not suggest that such tasks alone are sufficient to develop independent and confident mathematicians. Yet incorporating short tasks into the routine of lessons, particularly as a teacher new to the profession, was a highly convenient way for me to ensure that I provided the opportunities for discussion, critical thinking and creativity that my students deserved. For classes where group discussion or problem-solving is challenging for reasons of motivation, behaviour or prior attainment, short tasks offer a potential solution, providing an accessible means to promote the skills of evaluation, discussion and debate amongst students.

Indeed, the concept of a short task is adaptable. Certainly, some prompt is required, which seeks to force students to consider and evaluate the mathematics before them. Yet this could take a multiplicity of forms, provided it requires students to engage in critical and reasoned thought. Some end of term experimenting suggested that even frivolous questions can provoke interesting responses from students. When I asked my year seven age class which of the mean, median and mode would win in a fight, students argued passionately, ultimately settling on the mode as the likely victor because its popularity would ensure it always had friends to help it out. Whilst such questions may appear little more than end of term fun, they highlight the ways in which a careful consideration of task can shed new light on student understanding and encourage pupils to think creatively about the essential characteristics that underpin mathematical processes.

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**References**

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