Random thoughts

Tom Francome discusses the notion of randomness in mathematics education.

I am struck by the notion of randomness as both a mathematical concept and a pedagogical tool. In this article, I offer some thoughts on randomness in the classroom. I begin with some thoughts on mathematical randomness and then offer ideas on randomness as a pedagogical tool. I end with some thoughts on randomness in the organisation of teaching mathematics.

Mathematical randomness

There is much to be said for working explicitly on the nature of randomness with pupils. A possible mathematical definition of a random event is one such that every outcome is equally likely to occur. An example I might use in a classroom could be putting every pupil’s name into a hat and selecting one without looking. This is different to a perhaps more common usage of the word as something unusual or unexpected. Pupils might not think it very random if the same person were picked out of the hat three or four times in a row. This would be surprising but might be met with cries of “fix” if they have a sense of ‘equally likely’ meaning that each name should come up the same amount of times. This idea, like most misconceptions, comes from a logic that is worth celebrating but ultimately is something that students need to confront.

For me, one role of a teacher might be to engineer situations where conflicting ideas occur (within groups or individuals) so the differences can be worked on and new awarenesses arrived at through discussion. Finding a difference that makes a difference can be a good aim if you believe misconceptions should be worked on rather than avoided.

A familiar classroom task is the horse race game:

Everyone picks a number up to 12, then two dice are rolled and if the sum is your number you jump a fence. The first to, say, ten is the winner.

What is interesting for me about this activity is not just that it can be a way into comparing different ways of trying to list the outcomes and working out a best horse to pick, but also that if you pick the theoretical best, you do not always win. It can take a surprising number of trials to get a distribution close to that of the theory.

The standards unit (Swan, 2005) contains some well-thought-out statements to provoke discussion. For me, a worthwhile task for a mathematics teacher or department is to plan what questions or prompts you might offer if students have any of the misconceptions. For example, pupils often believe it is more difficult to roll a six on a single dice and we could understand why. I have been genuinely surprised by responses to the following, related, question:

On average how many rolls does it take to roll a six?

This is especially so when learners consider the mean and the median. I know if learners agree with the statement, “The probability of getting exactly three heads in six coin tosses is $\frac{1}{2}$”, I could set up a task to work on the ideas that you can estimate the probability of an event by conducting an experiment and see how this estimate improves with more trials.

Someone said, “the probability of getting 1 head when you toss 2 coins is $\frac{1}{2}$, so the probability of getting 3 heads when you toss 6 coins is $\frac{1}{2}$ because $\frac{3}{6} = \frac{1}{2}$”. Test this hypothesis by playing this game:

Flip six coins. You win (and draw a line to the right on the recording sheet) if you get exactly three heads. Otherwise, you lose (draw a line to the left on the recording sheet).

A Stirling recording sheet [http://nrich.maths.org/content/id/4304/RecordSheet.pdf](http://nrich.maths.org/content/id/4304/RecordSheet.pdf) can be a good way to record the trial data and develop awareness of more trials improving estimates of probability (see figure 1 p.21).
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Figure 1: A Stirling record sheet showing result of tossing coins experiment.

Pedagogical Randomness

John Mason has said that “habit forming can be habit forming” (Mason, 2002 p.141). Often when teachers talk about their practice, it is tempting to talk about things that “I always do”. Many of these practices might be useful to you as a teacher but I think the possibility of not doing what you always do affords some significant opportunities. This does require some work, as Mason explains including:

- noticing yourself doing something, noticing that you might act differently, noticing yourself about to do the thing but not aware of what to do differently and eventually noticing you are about to do something and having the option of not doing that… but something else (see p.74).

Attempting to do this can present a significant personal challenge. I will offer two ways that questioning your practice and using randomness might open up interesting new avenues.

When mathematics teachers plan lessons they often consider carefully the examples they will offer. Indeed, it is common in some countries with more prescriptive teaching than the UK for every student to meet some of the same examples. There is doubtless much merit in doing this as it is useful to think about what about an example is exemplary, whether it is an example of the same thing for the teacher as the pupils, what should be varied in the examples, what are easy examples and what are harder examples, what are non-examples and how you might use them and so on.

Perhaps, though, it is worth questioning what would happen if examples are not carefully chosen but random? Writing in MT107, Gattegno said all maths lessons should be shot through with infinity. Instead of one or two carefully chosen examples perhaps it is worth randomly generating lots of examples of a given type so pupils can come to know what changes and what is invariant by working with the infinity of examples. Consider the difference between:

- One given triangle on a page vs three points joined to make a triangle then animated randomly.
- An equation carefully designed by you to give a nice integer value vs randomly choosing four integers to go in an equation such as \( a \times b = c \).

Instead of making such an example using presentation software use formulae in a spreadsheet to generate examples. It is a possible advantage that these equations will not always have ‘nice’, small integer solutions. Dave Hewitt (1994, p.142) has made the point that teachers should “offer sufficient complexity for learners to have the material necessary to make use of their powers”. There is little point in pupils formally solving an equation when they can see the answer. A task like this might create a need for a more formal method. Learners could also be tasked with choosing numbers such that the solution is an integer, or a particular integer, or a particular fraction, offering a way for learners into making conjectures and into ‘being a mathematician’ (Francome, 2016).

Organisational randomness

A final aspect of randomness I wish to consider is that of organisational randomness. This could be related to the previous idea of questions being random. But, rather than generating the questions at random so they are unknown to both teacher and pupil, perhaps who gets to answer could be random? I have heard many well-meaning teachers talk about attempting to differentiate instruction by, “asking the low-attainers low-level questions and the higher-attainers higher-level questions”. There is a danger that this is exactly the type of practice that has the
potential to exacerbate any differences between pupils and confirm the teacher’s low expectations of particular pupils. As I have mentioned (Francome, 2014), every pupil needs challenging tasks so that they can make mistakes and learn from them. This includes being asked challenging questions and getting feedback on their thinking whatever their level of prior attainment. Human beings are not random, so a method of selecting pupils is required such as a random-name generator on the computer or, as Dylan Wiliam has advocated, lollipop sticks with pupils’ names written on. As well as potentially increasing equity, this has the added benefit of helping teachers make some of the most significant decisions in their day-to-day practice, namely whether or not to move on. It is all too easy to be lulled into a false sense of security if you ask a pupil who you expect to know the answer you want and they give it to you. They have said it, maybe others would have and, if not, at least they have heard the right idea so it is time to move on. At least if a student is selected randomly, you are more likely to get better information than by asking someone you expect to know.

This kind of organisational randomness can also inform decisions about who you talk to first when pupils are working. When I ask teachers how they decide who to talk to they normally give good reasons like, “Sarah because I noticed she was stuck on the starter” or, “Jane because she goes off task if you don’t check on her”. Perhaps these patterns should sometimes be interrupted? I was surprised and horrified by how little I knew about one pupil’s mathematics the first time I decided to randomly select the first five pupils I would visit in a lesson once they started work. Up until that point, I had thought I was well aware of my pupils’ needs and able to make good judgements about who I should intervene with but this practice helped me to reserve that judgement.

Organisational randomness can also be used to inform things like seating plans. I must have spent many hours over the years agonising over how to arrange pupils in a seating plan when I have decided we needed new arrangements. Much like ordering topics in a scheme of work, initially it seems easy but towards the end you can be left with a number of people that just do not work well together. Decisions around this might be about trying to support behaviour for learning, or getting a suitable mix of prior-attaining students working together or differing personalities. I was prompted to think about this after a session on mixed-attainment teaching at the ATM conference. I was reminded about when I first started teaching deliberately mixed groups and how I attempted to seat students according to their prior attainment. My classroom was arranged with desks in pairs and two pairs to make a group of four. I would try to arrange the seating so that each pairing was either a middle attainer/lower attainer or middle attainer/higher attainer but each four contained one of each pairing. This was because I assumed the ‘gaps’ should not be too wide. However, this was constraining and also operated on the faulty assumption that prior attainment told me anything remotely useful.

I later realised that a seemingly better solution was to randomise the seating generally and occasionally group students based on specific tasks. I was reminded of the advantages of randomising when reading about sampling for a research design course. Randomising the groups in your seating plan removes any systematic bias from the teacher’s choices but importantly allows you to randomise on both known characteristics, such as reading-age, as well as all sorts of unknown or unmeasurable characteristics like motivation, socio-economic group or how introvert/extrovert they are amongst other things. A similar argument can be put forward for how heads of mathematics decide on school groupings assuming they do not set by prior attainment.

I hope this article offers some ways of thinking about the potential role of randomness in your practice. Teaching is a complex activity and I think randomness might sometimes be used as a way of reducing some of that complexity for mathematics teachers.

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References


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