Everyone can be a mathematician

Tom Francome describes his mathematics department approach to ensuring all learners can become mathematicians.

I was recently invited to speak at the Birmingham branch meeting and at the ATM conference about some of the things that are unusual about our department. We teach in mixed attainment groups and work on only one theme each half term. I outline some of the reasons for this and some approaches (some new/some classic) that offer opportunities for anyone to think mathematically.

When I became a head of mathematics I found myself increasingly concerned that pupils were limited by the group in which they were placed. Teachers looked forward to teaching ‘top-groups’ because they got to teach interesting things to willing pupils and dreaded bottom-groups where they would drill ‘the basics’. In my opinion, there were too many pupils who did not enjoy mathematics either because it was boring and repetitive or because they were overwhelmed by the quick pace of lessons. I began to research the situation and found these concerns were shared by Boaler, Wiliam and Brown (2000) who suggested that a less procedural, time-pressured curriculum and heterogeneous grouping could help.

Six years ago, I resolved to change the way mathematics was taught in the school where I was head of mathematics. I wanted a scheme of work with space for pupils to explore and work on their own mathematics whilst developing a deeper understanding of content. I wanted a five-year curriculum where no pupils were limited by prior attainment. I wanted more pupils to enjoy mathematics, feel it was useful and feel they could succeed in the subject. I hoped that this could be achieved without a detrimental effect on results. I was influenced by Boaler’s The Elephant in the Classroom (2009) and Alf Coles’s session at the Birmingham branch of the Association of Teachers of Mathematics.

For me, many obstacles are removed if the focus is on ‘becoming a mathematician’ (Brown and Coles, 2008) as opposed to ‘learning mathematics’. It can also guide a clear way of working for teachers. Our goal is for all pupils to develop as a mathematician. I conjecture that no part of the curriculum content is as important as thinking for yourself, asking questions, making conjectures, being organised and systematic, recording and communicating what you notice, convincing, reasoning and explaining. Thinking mathematically is an entitlement of all pupils - not just those deemed ‘high-ability’. All pupils are capable of thinking hard about mathematics and entitled to interesting and varied opportunities to do so.

Making changes to teaching can present difficulties for teachers and, whilst there is evidence that setting has a detrimental effect, this could be a reason why so few schools teach mathematics to mixed attainment groups. I would like to talk briefly about the notion of ‘ability’ grouping.

Ability grouping depends on some underlying assumptions;

- Pupils have different ‘abilities’.
- These abilities can be known.
- Pupils will learn most effectively with others of similar ‘ability’.
- Self-concepts of low-attainers will be damaged by working with high-attainers.
- Teaching homogenous groups is easier.

I challenge these assumptions.

Research suggests ‘ability’ grouping has negative consequences for pupils across the attainment range (Boaler et al., 2000; Ireson et al., 2002b). High sets are taught in a high-pressure environment, at a fast pace where understanding is difficult. Teachers expect pupils to follow procedures, without detailed help or thinking time and teach as if the group is ‘one ability’, somewhere in the middle of the group’s attainment range. Low sets experience: low expectations; low level work that is boring or repetitive; less access to the curriculum; less opportunity for higher-order thinking; less discussion; less-experienced/non-specialist teachers; and they rarely move up a set (despite this often being described as the goal by many teachers) because they have not covered the same work.

My practice has been strongly influenced by Carol Dweck’s (2006) work on mindset. In a nutshell, people with a fixed-mindset believe their mathematics ability, say, is a fixed quantity they cannot change. People with a growth-mindset believe that ability can be increased with effort. Setting can put pupils (and teachers) into a fixed-mindset, which stops them from taking on challenging tasks and making mistakes they can learn from.
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It tells pupils:

“You’re good at mathematics… so you don’t have to try.”

or

“You’re not good at mathematics… so there’s no point in trying.”

There are also considerable social effects of ‘ability’ grouping. Meece et al. (1982) found a decline in girls’ conceptions of themselves as mathematicians despite higher achievement than boys. This decline led to reduced attainment later. Minority groups and disadvantaged pupils are over represented in low sets (Ireson et al., 1999). However, one of the most significant issues for teachers is that of attainment: the same pupil will do better if placed in a higher set than if placed in a lower set (Ireson, 2002b).

Many mathematics teachers have not considered teaching in mixed-attainment groups and those who have wonder legitimately: “So how do you teach mathematics to mixed groups?” Most people I talk to assume this will be much harder. It is still interesting and challenging and I am fortunate to have a hard-working team of teachers working with me but there are many advantages to teaching mixed attainment groups; a key one being that everyone is doing the same activities at the same time so teachers can talk to each other about what happens in their classrooms! This unusual idea allows sharing of planning, resources, stories, conjectures pupils have made or questions that may arise as well as good ways to support and challenge pupils. When I began teaching, I was always trying to come up with new tasks, now I find it useful to keep returning to the same tasks and being able to get more out of them.

Working with other teachers encourages you to be more sensitive in noticing desirable things happening so you can meta-comment, for example “here’s a good example of someone working as a mathematician, she’s trying all the possibilities systematically”. This allows you to make the things you think are important, important in your classroom. You praise the pupil whilst also highlighting what you value in the classroom (for instance, developing as a mathematician). In traditional classrooms where being good at mathematics means answering quickly or remembering the right method, fewer people can achieve. When there are more ways to be good at mathematics more people can be good at mathematics.

We tend to use a problem-solving approach where all pupils in the year do the same ‘common tasks’. Some pupils learn to like mathematics because they get things right. For me, the goal is to get stuck and try and get unstuck as that is an opportunity for learning to happen. Pupils need challenging tasks they can make mistakes with and learn from. Pupils need to work on their mathematics as well as mine/ours. We do six topics per year so we can work in depth on ideas rather than race ahead only to repeat the same things each year because pupils did not understand it.

I offer here four different ways we work mathematically with our pupils. We want activities which allow pupils to develop as a mathematician, solve problems collaboratively, discuss mistakes and misconceptions and work with learners’ ‘natural powers’. I will label the four types as: ‘Open-ended tasks’, ‘Open-middled tasks’, ‘Using mistakes and misconceptions’ and ‘Economic whole class teaching’.

Open-ended tasks

What we are looking for here are low threshold/high ceiling tasks where everyone can start but there are no limits on the levels of thinking that can be achieved. We want opportunities for pupils to make conjectures and ask/answer their own questions. As a space for exploring angles we might offer a 12-dot circular geoboard. There are great advantages of using the 12-dot geoboard and I was surprised how many people were unfamiliar with it but the task is simple:

![Join three points on the circumference with straight lines. What questions might a mathematician ask?](image)

The task can lead to some rich questions and the pupils can have a sense of ownership as the task develops. However, they will usually suggest ideas that you would be keen for them to work on as lots of nice bits of curriculum content come out. It is your decision as a teacher whether you get them all working on a particular question or allow them to work freely. The 12-dot geoboard is particularly nice from a teacher’s perspective as all possible angles are multiples of 15 so they can easily be checked for accuracy or if this fact is shared with pupils, they can then measure with confidence (if I’m unsure whether an angle is 74 or 75,
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I can ‘know’ it is 75). The geometry of the geoboard can be used to calculate any missing angles, which is a good opportunity for pupils to develop their reasoning skills. Conjectures can be explained or proven.1

Possible questions a mathematician might ask?

- What are the angles?
- How many different triangles are there?
- What are their angles?
- Can you sort them by type (isosceles, right-angled)?
- What do the right triangles have in common?
- What happens if you allow the centre to be used?
- Can you make any conjectures about the angle sums?
- What happens for more dots?
- What happens for more sides?

Open-middled tasks

Often when mathematics teachers ask questions, we are not that interested in the answers. I think of tasks as ‘open-middled’ if an answer exists but there is more than one way of reaching that answer and all of those methods are valued. Even with simple calculations, we are usually more interested in the process than the answer. It is often useful for teachers to ask a different question. What if not... “what’s the answer?” but “in how many ways can you get the answer?” A task I sometimes offer related to this concerns trapeziums:

Find as many different ways of calculating the area of a trapezium as possible.

If a group has never met this we might offer one way of finding the area and then allow them to explore others. The focus of the learning in the classroom is on the process, not the answer. We might get pupils to work in small groups to explain their methods clearly using diagrams or other representations. Groups need two things: group goals and individual accountability. One way of achieving this is to say that everyone in the group should be able to explain the methods so it is everyone’s responsibility to ask each other questions and make sure everyone else understands. We might choose someone at random to feedback on behalf of the group to encourage this. Possible support might be offered by providing some trapeziums on squared paper with several measurements given. As an extension pupils can prove algebraically why two methods are equivalent.

You might like to see how many you can come up with before looking at some examples of pupils’ methods at the bottom of the page.

Using mistakes and misconceptions

Some of the best classroom resources we have found are available from what we lovingly refer to as the ‘Big Blue Box’. That is the Standards Unit - Improving learning in mathematics.2 There are a handful of types of activity: classifying mathematical objects, interpreting multiple representations, evaluating mathematical statements, creating problems, analysing reasoning and solutions. These are based on extensive work led by Malcolm Swan who has been a big influence on my own work. The beauty of only having six types of activity is that pupils can get used to the type of activity so they can focus on doing the mathematics rather than learning the activity. A particular favourite task type of ours is evaluating whether statements are always true, sometimes true or never true. Statements are often made by pupils (and teachers) in mathematics classrooms. Often, a good question to ask is “is that always true, sometimes true or never true?”

Single statements such as:

- Two negatives make a positive.
- \(a^2 + b^2 = c^2\).
- \(x + x = 2x\).

can be used as assessment tools or a whole range of statements can be given and pupils can make a poster where they organise them under the three headings of always, sometimes or never true. It is a good activity from a planning perspective to think about what misconceptions there may be around a particular topic. An alternative is simply to write down conjectures the pupils have themselves made in a previous lesson to

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generate the material for a new activity:

I first came across this idea in the ATM publication *Thinkers*³, which is an excellent resource for easily developing your teaching through working on questioning.

**‘Economic’ whole-class teaching**

When we want to work on activities with the whole class at once ‘if we teach in a way that relies on the use of memory’ we may give the impression that mathematics is about remembering things quickly rather than thinking deeply. We choose instead, to try and work with awarenesses. Dave Hewitt (1994) offers some guidance into an approach for doing this by offering sufficient complexity, avoiding explanations, using imagery and the idea of successive levels of subordination. The ATM software *Grid Algebra*⁴ builds on the work of Gattegno and provides formal algebraic notation as feedback corresponding to movements on the grid. One move corresponds to one operation and the inverse movement is the inverse operation. Crucially none of these things need to be told to the pupils in order for them to know, and consequently they are able to develop ideas without relying on memory. Our experience has been that pupils are able to read and write formal notation in a much more sophisticated manner and work with greater ease on rearranging equations to solve them.

I hope these examples give a brief flavour of some ways of working with mixed attainment groups on developing their mathematics. Teachers’ beliefs are powerful and perhaps the most important aspect of teaching mathematics is for teachers to believe that any pupil can improve their mathematics with hard work, effort and good teaching and achieve at the highest levels. Everyone can be a mathematician, by doing the things that mathematicians do. No pupil should ever be limited by their prior attainment. Every pupil deserves the opportunity to think deeply about mathematics.

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At the time of writing he was Head of Mathematics at Kings Norton Girls’ School.

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**References**


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Footnote
1 http://nrich.maths.org/2883 has some excellent interactive and printable geoboard resources, and more ideas can be found in Geoff Faux’s excellent ATM resource, Exploring geometry with a 9-pin circular geoboard (2014).
3 Available from the Association of Teachers of Mathematics at www.atm.org.uk.
4 Available from the Association of Teachers of Mathematics.

Honorary Secretary's report

Every year the Honorary Secretary of the General Council writes a report for the Annual General Meeting which takes place at Conference. This is an abridged version of the report.

This is my final report as Honorary Secretary of the General Council (GC) of the ATM. It has been a great honour to serve the Association in this role. There were many successes during 2015. Conference was one of the most enjoyable and thought provoking I have attended. I was most excited by the number of young and early career mathematics teachers attending. The General Council is determined to find ways of engaging them and making the most of their energy and enthusiasm beyond the conference.

The website is under constant review and development. All past issues of MicroMath are available on the website alongside the complete archive of Mathematics Teaching. A new editorial team took over from MT250 and we hope that Mathematics Teaching continues to be a welcome arrival on your doormat or staff room table. Colleagues can now avail themselves of ‘membership anytime’ through the website and we have improved the search engine optimisation. The web team continue to enhance the members only area and GC reviews the website through monthly performance reports.

We hope you have noticed the increased use of social media to target individuals and organisations. Please retweet our tweets and ‘like’ Facebook postings. All publicity is vital to us in order to increase our profile and most importantly our membership. We have developed the group of ‘Outer Circle’ members and identified their areas of expertise and interest. If you are interested in joining the Outer Circle do get in touch.

Personal membership has risen and for the first time in 3 years income from subscription has risen slightly. We are particularly pleased in the rise of NQT membership. However, we still have a long way to go before we get the membership base that we need to sustain the Association. The GC has set a small deficit budget for the second consecutive year in order to fund the transition to a new website and to embed new office administration practices. We are confident we will be able to move into surplus budgets from next year. I must thank Kerry Belcher and the office team for the incredible work that they carry out on our behalf. The Association would not exist without them.

New publications during the year include Assessment in the new national curriculum: An ATM perspective; Talking maths; The algebra project; bigger ideas and Exploring area and fractions with a square geoboard. I hope that you all have your copies and that you are recommending these publications to your friends. The decision to provide a 24-hour free download opportunity of our assessment publication resulted in additional members. This shows that short-term marketing strategies can result in gains for the organisation. Future plans for the membership and marketing group include developing and trialling a mentoring scheme for NQT members with ‘Outer Circle’ volunteers.

I want to finish with a personal vote of thanks to Margaret Jones who has been editor of Mathematics Teaching for the last 4 years. She has been involved with the ATM for many years and has served as treasurer, led the workshop at conference and supported the Association in many other ways. GC decided it was appropriate to offer Margaret lifetime membership of the Association in recognition of the work she has carried out on our behalf.
The attached document has been downloaded or otherwise acquired from the website of the Association of Teachers of Mathematics (ATM) at www.atm.org.uk

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Mathematics Teaching does not seek to conform to an ‘official’ view on the teaching of mathematics, whatever that may be. The editorial board wishes to encourage contributors to express their personal views on the teaching and learning of mathematics.

ATM is an association of teachers in which everyone has a contribution to make, experiences and insights to share. Whether practical, political, philosophical or speculative, we are looking for articles which reflect on the practice of teaching mathematics. We aim to publish articles that will be of interest to the breadth of our membership, from the Foundation Stage to Higher and Further Education; as well as a balance between those derived from research and from practical experience. Submitted articles are accepted for publication based on their clarity, topicality, the extent to which they reflect upon knowledge and understanding of mathematics teaching and learning, and their contribution to inspiring further development and research.

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