Division in seconds by using multiplication in minutes

Ruth James elects to try an approach that sits outside her guiding principles in respect of teaching and learning

“I always hated times tables but now I love it!” … announced Clifton.

As a fun way of practising tables with my Year 5 class I have adapted ‘Minute Math’ an action research study of student self-assessment by Brookhart, Andolina, Zuza and Furman 2004, in which the children practise 100 times tables questions in 5 minutes. I believe that the children both enjoy this and that ultimately it works to embed times table knowledge.

After reading Division: What do we mean by ‘efficient methods’? (MT 239, Benson, 2014), who questioned whether the learner made appropriate method choices in solving division calculations, I was intrigued as to whether I could prove that a relationship between multiplication and division was seen by children as an element of an efficient strategy of their choice. In particular, “in order to achieve any kind of efficiency in calculation, a range of strategies and secure knowledge of multiplication tables are required”. (MT 239, Benson, 2014)

My pedagogical stance for mathematics learning is firmly rooted in building a community of learners within the classroom (MT240 How much Chocolate? Is 2/9ths more than 3/13ths?). I use mixed ability groups of four in all class teaching to encourage interaction and to promote the application of mathematical language to show reasoning. Every half-term the groups are changed to encourage diversity and fresh ideas from new learning partners. The sessions are structured to include my preferences of contextual problem solving, using visual images and manipulatives with an investigations bias. So, how does this approach, of a 100 times table randomly ordered question sheet, completed once a week by individuals, fit with a community of enquiry? I have reservations about the use of testing as a teaching strategy and the use of reward for attainment not effort. Using this out of context mathematical quiz (test!) on a regular basis goes against everything I strive for. Or does it? I decided to investigate what processes the children were undertaking to explain why, what appears to be learning through testing, was so successful.

I asked myself a serious question. Do I have to change my learning community principles? If not, how could I explain the 100 questions quiz in line with the principles of a community of enquiry?

How the quiz was organised

Every time the children arranged to complete the 5-minute quiz there was a huge buzz of excitement as individuals prepared to improve on their previous score. Each child had their own line graph to complete which consisted of two data points; a predicted score and on completion, an actual score. The children’s conversations asked what each other had done at home in preparation for the quiz that day.

Flora referring to times tables learning said …

“They (her parents) just say it (a times table question) and I write it!”

Other conversations included questions like …

“What did you get last week? What is your prediction?

Hufferd-Ackles, Fuson, Sherin (2004) suggest that peer questioning is a key component in a ‘Math-Talk Learning Community’. They advocate that within a maths community children increasingly take responsibility for their own learning and for self-evaluation and the evaluation of others.

In Figure 1 Joe has only attempted the times tables questions he feels proficient enough to answer accurately. After peer marking, Joe could clearly see which times tables he needed to practise in order to increase his score next time. The teaching instruction was to beat your last score by at least one percent to earn a house point, as an incentive to all rather than a reward for the highest attainment. An equal score did not gain a house point! Dweck, (2006) showed through her research that children with a growth mindset were able to increase their scores by “mobilising their resources for learning” (page 58) by practising and believing that they could achieve through their own efforts.

My Year 5 children liked the idea that they were rewarded for effort even though they had not achieved the 100% goal. Equally they were disappointed, but not devastated, if they didn’t achieve better than last time. This was not seen as a negative reinforcement, as it spurred them on to do better subsequently. The children were clearly setting their own targets and enjoyed adding to
the graph, thus improving their data representation and graphing skills, see Figures 3 & 4. Pupils knew the aim was for an accurate prediction to show a self-assessed understanding of their own performance. William (2011) believes that self-assessment improves learning through self-regulation, that is children setting their own goals. The idea of metacognition (Flavell, 1976) includes knowing what you know (metacognitive knowledge), what you can do (metacognitive skills) combined with what you know about your own learning and cognitive abilities (metacognitive experience). William suggests that children are motivated to use these ideas when they can see the value of the consequences. I suggest that for the children in this study setting their own goals before, or adjusted during the quiz, made sense of their learning to them.

“I've practised this week, and I think I'm ready!”

April, “I've been practising. I'm going to get 90 or 95!” or alternatively

“Oh no, I've forgotten to look up my 6’s!”

The children's perception of this learning was not competitive between peers, suggesting a more level playing field where they had the same improvement expectation outcome. Most children accepted the inevitability that if they hadn't practised, they expected their score to be lower. However this was not always the case, which convinced some children that the more effort they applied during the quiz itself in addition to their endeavours before the quiz, the better their result might be. Moreover, their comments did show the way the children felt about their prediction ultimately aiming to beat their previous score. In this way the classroom formalised the process of repetitive learning from home: many children included comments such as

“My Mum will be pleased if I can beat my score after last week!” Joe said, showing support for practising from his mother which confirmed a real link between school and home.

The quiz itself lasted 5 carefully timed minutes, when silence ensued. This was a great assessment opportunity to see which children were still relying on fingers for sequential support or which organisational strategies they had employed. Following the quiz, the children peer marked their papers, either from a completed example or as a group, verbally listing the answers by columns. Adrian said as he was adding up the columns of correct answers ...

"Whew! I'm doing maths now "

I interpreted this to mean that he recognised that mental recall of the quiz (test) was only a means to an end and not the mathematical problem of finding an accurate score for his learning partner!

The discussion that followed the marking was where the real community of learners was revealed. Despite each child aiming through showing a mastery orientation by aiming for a perfect result, the interaction included mathematical dialogue about scores, strategies for success and congratulating others. The children's control over their learning following self-assessment was evident in their verbal comments. For example, Eliza hadn’t answered any eight times table questions and so this indicated her next steps in learning to improve her score next time.

The Result?

Figure 2: Joe – Multiplication week 10 (90%)

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Figure 3: Flora – Multiplication weeks 1-7

Children loved the process of prediction as anticipation and scoring, within a community, regardless of their score. In particular they liked to watch their own and their peer's progress grow and many didn’t join the predicted dots on the graph to not detract from the actual result, see Figures 3 and 4. The children who achieved quickly were willing to share their ideas and praise from other children over a new idea or technique.

Nick said … “When you get a good score you still want to do better”, Alexa said … “It shows in the weeks how much progress I make”, and Joe said … “The graph encourages me to get higher than my prediction and if I am, I’m making progress”.

Brookhart, Andolina, Zuza and Furman (2004) advocate a written reflection after each test. However, my previous experience has shown that a written reflection can be worthless, as it appears to demean any achievement in
the eyes of the children, as they still have to fill the sheet regardless of how well they do. Verbally expressing their delight to a partner, combined with showing their strategies of all columns first, or all rows, or “the ones I know best first”, has a greater impact on the children in helping them to remember their strategy, their successes, and their failures for next time.

Those children who achieved 100%, 6% of children after 3 weeks and 20% of children after 6 weeks, completed an extension set of questions to apply their times table knowledge to 30 double-digit x double-digit questions. In Figure 5 Maria chose a combination of mental and written methods based on efficiency in order to achieve 29/30 in five minutes.

Children with special needs worked on different simple addition, subtraction and multiplication questions, starting with 20 questions in 5 minutes and gradually increasing to 50 questions in the same time, at the end of the study.

Amy said … “I like doing it slowly, it makes me think hard and not struggle.”

This group of children explained that they felt part of the learning community as they marked their learning together and shared their successes with the class.

What next?
The National Curriculum (2013) considers practise to be a significant part of learning mathematics, so I wondered what might be a worthwhile extension after the initial 13 weeks. I asked the children if they enjoyed the quiz and was assured that they did.

Louisa wrote … “I look forward to 5-minute maths because I’m getting better every time!”

Laurence said … “I think it’s amazing because it helps me so much.”

Therefore, the next six weeks saw the same process but with 100 division questions, with products divided by one of the factors without remainders. The children approached this task again with familiarity, with expectation, and perhaps with some anxiety over the word DIVISION! Once again this proved very successful with most children out performing their times tables results.

John wrote … “I really like division because I got a lot higher scores in division than multiplication and I really enjoy it!”

This comment reinforced how John felt about himself and his positive identity. The other children also recognised that John had improved over time and were keen to share his success by commenting on his line-graph individually and as a class community.

I asked the children through a short questionnaire which they found easier in 5 minutes, division or multiplication?

Emelia wrote … “division because it’s easier to count up”

… perhaps referring to a mental visualisation of a number-line and grouping indicating a multiplicative link, such as skip-counting, showing additive reasoning but understanding inverse relations between operations.

Evangeline wrote … “division is easier because it’s just times tables. I like division because I know my times tables”

… clearly Evangeline recognises the inverse link and can engage with the questions and apply her knowledge.

Of the 25 children that were asked, 11 children preferred multiplication, 6 children preferred division, and 8 children chose both! I found this surprising that more children didn’t chose multiplication as this has been the focus of much mathematical recall teaching in Key Stage 2. I was also encouraged as during the final five weeks the children worked on a combination of 100 multiplication and division questions.
Can the children apply their knowledge?

To test my hypothesis of learning progress through practise and repetition, I asked the children to complete a questions sheet with questions taken from Benson’s article. I adapted the recording to show the following alternatives as a calculator was not an option.

Some children could clearly see how their times table and division knowledge could help them solve non-contextual problems within the short time allowed, suggesting fluency. However, the key element here was the discussion approach to problem solving and reasoning, in their groups of four, about which answers were correct and why. Maria explained that she had used a combination of mental recall of times tables, written number sentences, the compact method of subtracting multiples, and diagrams to complete the paper. Franco wrote at the end of his paper … “I love division because it’s a challenge for me and I love challenges”. He recognised that he had found the questions tricky relying on peer dialogue for support about choosing a written method … number-line, chunking, or bus stop, but not the answers!

Children attempted nearly all the questions and expressed their ‘ability’ to complete the questions mainly “in my head” as a positive response, associated with the 5-minute maths quizzes. I believe that this scenario suggests that fluency and recall help to confirm ownership of children’s own progress within a community of learners. There was no expectation of a formal procedure and therefore a combination of written jottings with mental recall of tables for the compact method of subtracting multiples, and the use of known facts for estimation, enabled children to complete the paper. To consider one of the aims of the new National Curriculum:

“… become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.”

In association with my initial question whether I could prove that a relationship between multiplication and division was seen by children as an element to consider when choosing an efficient strategy. I believe that the children in my class did choose an efficient strategy in approaching the problem, applying their learning - see Figure 8, which was to estimate first and to use mental methods before attempting any written algorithm. My simple quiz helped children become fluent, through frequent if not altogether varied practise to embed one of the fundamentals of mathematics – multiplication facts. Despite what looks like teaching through repetition of testing, the community dialogue was the key element in this learning. Without peer support, a risk taking culture, such as peer marking along with sharing of scores, and with a collective mathematical understanding of the need for accurate recall of mathematical facts to help solve associated problems, I suggest that this practice may have had a different impact.

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References


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