More Vedic Mathematics activities for the classroom

Activity 2

Below there are four examples of multiplying special numbers. Look at these examples, and then answer the questions that follow:

12·18=?
(i) 1·2=2
(ii) 2·100=200
(iii) 2·8=16
(iv) 200+16=216 ⇒ 12·18=216

27·23=?
(i) 2·3=6
(ii) 6·100=600
(iii) 7·3=21
(iv) 600+21=621 ⇒ 27·23=621

75·75=?
(i) 7·8=56
(ii) 56·100=5600
(iii) 5·5=25
(iv) 5600+25=5625 ⇒ 75·75=5625

99·91=?
(i) 9·10=90
(ii) 90·100=9000
(iii) 9·1=9
(iv) 9000+9=9009 ⇒ 99·91=9009

Questions:
(1) What is special about the numbers that appear in each product?
(2) Which of the following products can be calculated using the method presented in these examples: 36·35 ; 68·62 ; 97·83. Why?
(3) Describe, in your own words, the rule underlying the Vedic method for multiplying these numbers.
(4) Use the method to calculate 86·84. Verify your answer by using a calculator.
(5) What is the mathematics that underlies the Vedic method?
Look at the second example: 27·23
Instead of 27 we can write: 2·(2+1)
and instead of 23 we can write: 2·10+3.
Thus:
27·23 = (2·10+7)(2·10+3) = 2·10·2·10 + 2·10·3 + 2·10·7 + 7·3 =
= 100·2·2 + 2·10·(3+7) + 7·3 = 100·2·2 + 2·10·10 + 7·3 =
= 100·2·2 + 100·2 + 7·3 = 100·2(2+1) + 7·3 = 100·[2(2+1)] + 7·3
Look at the brackets [2·(2+1)] and 7-3. What is their connection to the method presented above?

(6) Look at the fourth example: 99·91
Instead of 99 we can write: _______.
Instead of 91 we can write: _______ = _______.
Repeat the process presented in (5), and explain why the method is appropriate for calculating 99·91.

(7) Try to verify the method for any multiplication of two 2-digit numbers having the same tens-digits and the sum of their units-digits is 10, by providing a general proof.
(8) In which part of your proof did you use the fact that the tens-digit of both numbers is identical? In which part of your proof did you use the fact that the sum of the units-digits of the two numbers is 10?
(9) Find two numbers that can be multiplied using the method shown in Activity 1 and the method shown in this activity. Which of the methods is easier to use in this case? Why?
Solutions to Activity 2
The four examples demonstrate a multiplication of two 2-digit numbers having the same tens-digit and the sum of their units-digits is 10.
The following is a literal description of the rule underlying the Vedic method for multiplying such numbers:
(i) Multiply the number that represents the tens digit by its successive number;
(ii) Multiply the result obtained in step (i) by 100;
(iii) Multiply the units-digit of both numbers;
(iv) Add the results obtained in steps (ii) and (iii).

Question (5) shows students how to make sense of the Vedic method.
In order to verify the Vedic method, as asked in question (7), let $10a + b$ and $10c + d$ be the two numbers we wish to multiply. As both numbers have the same tens-digit, they can be written as $10a + b$ and $10a + d$.
Therefore:
$$(10a + b) \cdot (10a + d) = 10a \cdot 10a + 10a \cdot d + 10a \cdot b + b \cdot d$$
Performing algebraic manipulation on this expression in order to better understand its meaning gives:
$$10a \cdot 10a + 10a \cdot d + 10a \cdot b + b \cdot d = 10a \cdot 10a + 10a \cdot (d + b) + b \cdot d$$
However, as asked in question (8), since $d + b = 10$, we obtain:
$$10a \cdot 10a + 10a \cdot (d + b) + b \cdot d = 10a \cdot 10a + 10a \cdot 10 + b \cdot d =$$
$$= 100a \cdot a + 100a + b \cdot d = 100[ a(a + 1) + b \cdot d$$

It should be noted that the above proof verifies the method for any two numbers, not just 2-digit numbers, having the same first-digits and where the sum of their units-digits is 10, since any number can be written in the form $10x + y$. However, in cases where the numbers are larger than 100 the computations might not always be as simple.

For example, in order to calculate: $502 \cdot 508$, the following steps are required:
(i) $50 \cdot 51 = 2,550$ ; (ii) $2,550 - 100 = 255,000$ ; (iii) $2 \cdot 8 = 16$
(iv) $502 \cdot 508 = 255,000 + 16 = 255,016$.
Advanced students may deduce this fact with suitable probing questions.
In order to answer Question (9) students have to find the common characteristics of the numbers that appear in activities 1 and 2. The numbers should have the following features: be 2-digit numbers; be close to a power of 10, in this case $10^2$; due to the constrains imposed by Activity 2; and have the same tens-digit, and the sum of their units-digits be 10. There are several numbers that can be multiplied through implementing both methods, for example: $87 \cdot 83$ ; $96 \cdot 94$ ; $95 \cdot 95$.
Calculating $96 \cdot 94$ using the Vedic method shown in Activity 1 (in the journal) gives:
$$96 \cdot 94 = 10^2 \cdot [(10^2 - 4) - 6] + 6 \cdot 4 = 100 \cdot 90 + 24 = 9,024$$
Calculating $96 \cdot 94$ using the Vedic method shown in Activity 2 gives:
$$100 \cdot [9 \cdot (9 + 1)] + 6 \cdot 4 = 100 \cdot 90 + 24 = 9,024$$
Students may be encouraged to discuss the differences between these two methods and to explain any preferences they may have.
Activity 3

Below are three examples of multiplying special numbers. Look at these examples, and then answer the questions that follow:

\[
\begin{align*}
12 \cdot 92 &= (i) 1 \cdot 9 + 2 = 11 \\
& \quad (ii) 11 \cdot 100 = 1,100 \\
& \quad (iii) 2 \cdot 2 = 4 \\
& \quad (iv) 1,100 + 4 = 1,104 \\
& \Rightarrow 12 \cdot 92 = 1,104 \\
23 \cdot 83 &= (i) 2 \cdot 8 + 3 = 19 \\
& \quad (ii) 19 \cdot 100 = 1,900 \\
& \quad (iii) 3 \cdot 3 = 9 \\
& \quad (iv) 1,900 + 9 = 1,909 \\
& \Rightarrow 23 \cdot 83 = 1,909 \\
76 \cdot 36 &= (i) 7 \cdot 3 + 6 = 27 \\
& \quad (ii) 27 \cdot 100 = 2,700 \\
& \quad (iii) 6 \cdot 6 = 36 \\
& \quad (iv) 2,700 + 36 = 2,736 \\
& \Rightarrow 76 \cdot 36 = 2,736
\end{align*}
\]

Questions:

(1) What is special about the numbers that appear in each product?

(2) Which of the following products can be calculated using the method shown in these examples: 58 \cdot 48 ; 68 \cdot 62 ; 97 \cdot 17. Why?

(3) Describe, in your own words, the rule underlying the Vedic method for multiplying these numbers.

(4) Use the method to calculate 47 \cdot 67. Verify your answer by using a calculator.

(5) What is the mathematics underlying the Vedic method? Look at the third example: 76 \cdot 36

Instead of 76 we can write: 7 \cdot 10 + 6, and instead of 36 we can write: 3 \cdot 10 + 6.

Thus:
\[
76 \cdot 36 = (7 \cdot 10 + 6)(3 \cdot 10 + 6) = 7 \cdot 10 \cdot 3 \cdot 10 + 7 \cdot 10 \cdot 6 + 3 \cdot 10 \cdot 6 + 6 \cdot 6 = \\
= 100 \cdot 7 \cdot 3 + 10 \cdot 6 \cdot (7 + 3) + 6 \cdot 6 = 100 \cdot 7 \cdot 3 + 10 \cdot 6 \cdot 10 + 6 \cdot 6 \\
= 100 \cdot 7 \cdot 3 + 100 \cdot 6 \cdot 6 + 6 \cdot 6 = 100 \cdot 2(2 + 1) + 6 \cdot 6 = 100 \cdot (7 \cdot 3 + 6) + 6 \cdot 6
\]

(6) Repeat the process described in (5) to calculate the product 23 \cdot 83.

(7) Try to verify the method for any multiplication of two numbers having the same units-digits and where the sum of their tens-digits is 10, by providing a general proof.

(8) In which part of your proof did you use the fact that the units-digit of both numbers is identical? In which part of your proof did you use the fact that the sum of the tens-digits of the two numbers is 10?

(9) Find two numbers that can be multiplied using the method shown in Activity 2 and the method shown in this activity. Which of the methods is easier to use in this case? Why?

(10) Can you find two 2-digit numbers that can be multiplied using the method that appears in Activity 1 and the method shown in this activity? Why?

Solutions to Activity 3

The three examples demonstrate a multiplication of two 2-digit numbers having the same units-digit and where the sum of their tens-digits is 10.
The following is a literal description of the rule underlying the Vedic method for multiplying such numbers:

(i) Multiply the tens digits of the two numbers, and add the common units digit;
(ii) Multiply by 100 the result obtained in step (i);
(iii) Multiply the common units digits;
(iv) Add the results obtained in steps (ii) and (iii).

In order to verify the Vedic method, as asked in question (7), let \(10a + b\) and \(10c + d\) be the two numbers we wish to multiply. As both numbers have the same units digit, they can be written as \(10a + b\) and \(10c + b\). Therefore

\[
(10a + b) 
\times (10c + b) = 10a \cdot 10c + 10a \cdot b + 10c \cdot b + b \cdot b
\]

In order to better understand the meaning of this expression, algebraic manipulation gives:

\[
10a \cdot 10c + 10a \cdot b + 10c \cdot b + b \cdot b = 100a \cdot c + 10b \cdot (a + c) + b \cdot b
\]

However, as asked in question (8), since \(a + c = 10\), we obtain:

\[
100a \cdot c + 10b \cdot (a + c) + b \cdot b = 100a \cdot c + 10b \cdot 10 + b \cdot b
\]

\[
= 100(a \cdot c + b) + b \cdot b
\]

In order to answer Question (9) we have to find the common characteristics of the numbers in activities 2 and 3. The product 55-55 is the only one that meets the requirements for both activities.

Calculating 55-55 using the Vedic method shown in Activity 2 gives:

\[
100 \times 55 = 55 \times 55 = 3,025
\]

Calculating 55-55 using the Vedic method shown in Activity 3 gives:

\[
100 \times 55 + 5 \cdot 5 = 3,025
\]

Obviously, in this case the calculations are almost identical, as it is easy to see that

\[
5 \cdot (5+1) = 5 \cdot 5 + 5
\]

In order to answer Question (10) we have to find the common characteristics of the numbers in activities 1 and 3. The numbers should have the following features:
be 2-digit numbers; be close to a power of 10 (in this case \(10^2\)); have the same units digit; and the sum of their tens-digits is 10. The number 97 is an example of a number that meets the first two requirements. In order to meet the last two requirements we can only chose 17 to be the other number in the product. However, 17 is not close to 100, and therefore does not correspond the second characteristic. Generally, if one of the numbers in the product is larger than 80 and smaller than 100, in order to meet the first two requirements, then the other number should be less than 30, and therefore it is not possible to find two numbers that can be multiplied using both methods.
Activity 4
In the following examples we have the Vedic method for squaring 2-digit numbers. Look at these examples, and then answer the questions that follow:

<table>
<thead>
<tr>
<th>The number</th>
<th>Steps</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>11²=?</td>
<td>(i) 11+1=12</td>
<td>(ii) 12·10=120</td>
</tr>
<tr>
<td>12²=?</td>
<td>(i) 12+2=14</td>
<td>(ii) 14·10=140</td>
</tr>
<tr>
<td>14²=?</td>
<td>(i) 14+4=18</td>
<td>(ii) 18·10=180</td>
</tr>
<tr>
<td>25²=?</td>
<td>(i) 25+5=30</td>
<td>(ii) 30·20=600</td>
</tr>
<tr>
<td>36²=?</td>
<td>(i) 36+6=42</td>
<td>(ii) 42·30=1260</td>
</tr>
</tbody>
</table>

Questions:
(1) Describe, in your own words, the rule underlying the Vedic method for squaring 2-digit numbers.
(2) Use the method to square 18, 27, and 42. Verify your answer using a calculator.
(3) What is the mathematics underlying this Vedic method?
   Remember that squaring a number means multiplying the number by itself. Look at the fourth example. Instead of 25 we can write: 20+5. Thus:
   \[ 25^2 = (2 \cdot 10 + 5)^2 = (2 \cdot 10 + 5) \cdot (2 \cdot 10 + 5) = 2 \cdot 10 \cdot 2 \cdot 10 + 2 \cdot 10 \cdot 5 + 5 \cdot 5 \]
   We now manipulate this expression to give:
   \[ 2 \cdot 10 \cdot 2 \cdot 10 + 2 \cdot 10 \cdot 5 + 5 \cdot 5 = 2 \cdot 10 \cdot (2 \cdot 10 + 5) + 5 \cdot 5 \]
   Can you see the connection between this expression and the Vedic method?
(4) Repeat the process described in (3) to square 36.
(5) Try to verify the Vedic method for squaring any 2-digit number by providing a general proof.
(6) Can the Vedic method be used for 3-digit numbers as well? Try to use the method to calculate 123².
(7) Give an example of a number that can be squared using the method shown in Activity 1 and the method shown in this activity. Which of the methods is easier to use in this case? Why?
(8) Find a number that can be squared using the methods presented in Activity 2 and Activity 3, and in this activity. Which of the methods is easier to use in this case? Why?

Solutions to Activity 4
In this activity it is assumed that students are familiar with the concept of squaring a number as multiplying the number by itself, and are not yet acquainted with the short multiplication formulae.

The five examples demonstrate the Vedic method for squaring 2-digit numbers. The following is a literal description of the rule underlying the Vedic method:
(i) Add the units-digit to the given number; (ii) Multiply the result obtained in step (i) by the tens-digit of the given number; (iii) Square the units-digit; (iv) Add the results obtained in steps (ii) and (iii).
In order to verify the Vedic method, as asked in question (5), let \(10a+b\) be the number we wish to square. Squaring the number we obtain:

\[(10a+b)^2 = (10a+b)\cdot(10a+b) = 10a\cdot10a + 10\cdot a\cdot b + b^2\]

Manipulating this expression to better understand its meaning gives:

\[10a \cdot (10a+b+b)+b^2 = 10a \cdot [(10a+b)+b]+b^2\]

Referring to question (6), it should be noted that the general proof above indicates that the Vedic method for squaring numbers can be applied for numbers with more than 2-digits, where \(a\) represents the digits of the number to be squared without the units-digit, \(b\). However, in such cases the calculations are more complicated, as they require repeating the above steps several times, depending on the number of digits in the number we wish to square.

For example, squaring 123 necessitates implementing the following:

(i) \(123+23=146\); (ii) \(146\cdot100=14,600\)

(iii) We now have to square 23, and therefore we have to repeat the Vedic method:

\[23^2: (i) \ 23+3=26; (ii) \ 26\cdot20=520; (iii) \ 3^2=9; \ 520+9=529\]

Therefore: \(123^2=14,600+529=15,129\)

Squaring 352, for example, necessitates implementing the following:

(i) \(352+52=404\); (ii) \(404\cdot300=121,200\);

(iii) \(52^2=\) ?

(i) \(52+2=54\); (ii) \(54\cdot50=2,700\); (iii) \(2^2=4\)

Therefore: \(352^2=121,200+2,700+4=123,904\)

In order to answer Question (7) we have to find the common characteristics of the numbers that appear in Activities 1 and 4, namely: 2-digit numbers close to 100.

Taking 98 as an example, and employing the method presented in Activity 1 we get:

\[100\cdot[(100-2)-2]+2\cdot2=9,600+4=9,604\]

Using the method shown in Activity 4 we get:

\[90\cdot(98+8)+8^2=90\cdot106+64=9,540+64=9,604\]

Obviously, in this case the method shown in Activity 1 is much easier to use. Students may conclude that the method that appears in Activity 4 is not always simple, and it is dependent on the number to be squared. Generally, squaring numbers close to a power of 10 can be done more easily using the method shown in Activity 1.

Comparing the methods shown in Activities 2, 3, and 4, in response to question (8) gives:

Activity 2: \(55^2 = 55\cdot55=100\cdot[5\cdot6]+5\cdot5=3,000+25=3,025\)

Activity 3: \(55^2 = 55\cdot55=100\cdot(5\cdot5+5)+5\cdot5=3,000+25=3,025\)

Activity 4: \(55^2 = 50\cdot(55+5)+5^2=3,000+25=3,025\)

As can be seen, in this case all three methods are easy to use, and preference for one particular method is for students to discuss. It should be noted that 55 is incompatible with the requirements included in Activity 1, and therefore it is not useful to use the method shown in Activity 1 to square 55.