TEACHING LEVEL 6 MATHS TO YEAR SIX
Rachel New explains her approach to the challenge of working with more able pupils

For the last two years, I have had great fun teaching Level 6 Mathematics to a small group of Year 6s, 8 last year and 11 this, for two hours every Friday afternoon for a ten week block before the SATs. Most of my experience is as a secondary school teacher, and teacher trainer, and I jumped at the chance in particular to introduce basic algebra to some tabulae rasae uncorrupted by keen parents teaching old-fashioned methods to their under-stimulated children! In this article I share some thoughts and ideas about how this might work best. Many of the ideas could also work as extension tasks within a mixed ability Year 6 class. The aim was for pupils to enjoy being stretched, rather than to coach them to get a level 6 in the SATs. In particular, I wanted to focus on their problem solving skills, in making connections between different mathematical concepts and in developing logical ways of showing working. It was fun to go off at a tangent sometimes, for example to explore what happens when you multiply or divide by smaller and smaller numbers.

It was clearly impossible to cover all topics in this time frame, and I wanted to get away from scheme-of-work driven lesson planning as much as possible. Roughly, I spent the first two weeks, on basic algebra; then two on area, perimeter, volume and transformations; two on data handling, specifically probability, scatter graphs, averages and range; one on percentages, fractions, decimals, and ratio although these came up throughout; two on graphs and equations; and one on exam paper feedback. How to make exam paper feedback interesting and useful is perhaps the subject of a future article! Learners were given SATs questions for homework on the topic covered, and there were always packs of extra questions on miscellaneous topics available for extension tasks for the one or two pupils who finished the class tasks quickly. Covering less topics more thoroughly is a deliberate strategy, especially since the level 6 papers require very few new methods or prior technical knowledge, apart from the conventions of algebra, and are more about creative problem solving and understanding concepts. Examples include solving problems with two way tables; working out data items from clues about the mean; finding a value for \( x \) when an expression equals a prime number; changing a decimal into a fraction out of 200; 10% of \( \frac{1}{2} \); and creative ways of finding compound areas. Explaining why answers or methods are wrong is a good way to encourage conceptual thinking and the use of estimation to check calculations. Scientific calculators were used, necessary for using \( \pi \) and for the order of operations, as well as powers and roots, which pupils enjoyed investigating. The children could also explore rules for negative numbers on calculators as an extension task.
In this article I focus on algebra, and then on some general principles for test performance.

**Introducing algebra**

I love teaching algebra for the first time before anyone has had a chance to corrupt learners with “rules” too early on and simplifying exercises that are not in context and have no concrete experiences behind them. Children at this, or any, age need to generalise from what they already know about numbers, and to have some motivation to use algebra. After showing them lots of slides illustrating how algebra is used in a wide range of jobs, with some very intimidating equations!, I began with the following activities, which I called “How numbers behave”:

1. **True (stand up) and false (sit down) for equations such as**
   
   \[5 \div 10 = 10 \div 5 \] and \[8 + 9 + 10 = 10 + 9 + 8\]
   
   After examples that involved numbers, I tried the same with letters for example,
   
   \[a \times b \times c = c \times b \times a\]

2. **Pupils were then able to generalise first with words and then using letters from several arithmetical examples of each of these four “rules”:
   
   **Conventions such as** \(3 \times a\) **for** \(3 \times a\) **etc were introduced a little later in the lesson. The point of algebra here is as shorthand for long sentences such as “When adding two numbers, you can swap them round”. I used mini whiteboards extensively for all these activities, which are ideal for picking up misconceptions early on. My hypothesis is that once a mistake has been made twice it becomes embedded in the memory and is very hard to replace! This is one reason why many short active tasks on the same concepts are vital when introducing algebra.**

3. **I then gave learners many slides like Figure 6 where they had to pick out expression(s), which I defined as calculations without answers, to match the words at the top.**

   ![Figure 6](https://via.placeholder.com/150)

   **When you are adding, you can swap the numbers round.**
   
   \[a + b = b + a\]
   
   **When you are subtracting, you can’t swap the numbers round.**
   
   \[a - b \neq b - a\]
   
   **When you are multiplying, you can swap the numbers round.**
   
   \[a \times b = b \times a\]
   
   **When you are dividing, you can’t swap the numbers round.**
   
   \[a \div b \neq b \div a\]

   **Figure 5**

   ![Figure 7](https://via.placeholder.com/150)

   **Figure 7**

   **Figure 8**

   **Dice Games 1**

   ![Dice Games 1](https://via.placeholder.com/150)

4. **We played many dice games involving \(d\) as the number on the dice and the value of the expression as your score, such as Figure 8. From this context, it was possible to ask questions like “My score is always \(20 - 2d\). If I throw a 3, what is my score?” but also “My score is always \(7d + 1\). If I score 36, what must have I thrown on the dice?” This eventually leads to “What is the value of the expression \(20 - 2d\) when \(d = 3\)” and “Solve \(7d + 1 = 36\)” but I didn’t use that vocabulary until pupils had experienced the processes in context.**

5. **Learners took part in races with music where cards with expressions stuck on the walls around the room had to be grabbed and stuck on the board to match the words, for example “four more than the dice number”. Two sets of the same cards, in different colours, are stuck on the walls and two pupils are chosen to take part, competing against each other to get the card in the right place on the**
I did not introduce any powers until about Week 7, as experience shows that children will use them all the time from then onwards! Here are basic and advanced versions of the race:

Over the weeks we did many starters on generating and substituting into expressions, all in the dice game context. Regular expansion of for example, $3d$ as $d + d + d$, rather than just $3 \times d$, helped to avoid getting confused with $d^3$ and $d + 3$, etc. Use of number machine chains to form expressions was a good way to understand the order of operations, especially brackets. There was an interesting discussion on why there could not be a square root of $-1$.

Forming expressions from "Think of a number" problems is a valuable introduction to proof, as well as providing a purpose to forming expressions. I first gave learners the instructions on the left and left the mystery of how it works hanging in the air. Later in the lesson I got them to form the expressions on the right to see how algebra can explain things. Children will enjoy making up their own, either to end with the number they started with, or to end with the same number every time.

Equations: what are they?

To begin, I like to give the "big picture" of the concept of an equation. So many teachers I have observed dive straight into how to solve an equation without talking about what an equation is, or what it means to solve one. Figure 14 picks up on an illogical way of showing working that I put right in the first week for example, writing $3 \times 4 = 12 + 2 = 14$ instead of either splitting this up into two separate calculations using two lines of working such as $3 \times 4 = 12$, and then $12 + 2 = 14$ on the next line, or by using brackets $(3 \times 4) + 2 = 12 + 2 = 14$. 
Next, I encourage trial and improvement to get a feel for what it means to solve equations. Of course solving simultaneous equations formally is a level 7 skill, but they have appeared in the level 6 KS2 SATs and can be solved without formal methods. In the first one below, if pupils need a hint, I ask what the difference is between the first and the second equation such as the first is $x$ more than the second, so $x$ must be the difference between 11 and 7. Providing hints is a good way to differentiate: you may have one or two pupils who are confident enough to work without any hints, but others can ask for the hint after having a try on their own first. See also Figure 2: you can remove the two hints to start with.

Equations with the unknown on both sides: trial and improvement

If there is time, or as an extension task for this lesson, pupils try out different values for the unknown in a linear equation with the unknown on both sides, a method of trial and improvement, again to get a feel for what it means to solve an equation by focusing on the goal of making the left-hand side of the equation equal to the right-hand side:

Solve $3x + 31 = 8x + 6$

Try $x = 1$ LHS = 34 RHS = 14 LHS ≠ RHS (and gap of 20 between them)

Try $x = 2$ LHS = 37 RHS = 22 LHS ≠ RHS (but the gap is only 15 now)

Even less able pupils will develop an intuitive ability to decide whether to try something bigger or smaller next, and enjoy this kind of approach. Ideally, one would then teach the balancing method of solving linear equations, but I think this is best left until Year 7 onwards as it needs a great deal of practice to avoid embedding mistakes, in line with my hypothesis above.

Using psychological principles to enhance test technique and problem solving skills

When it came to the tests themselves it was important to remember, as one parent commented afterwards, that they are still children. I gave each pupil a cuddly toy to stroke during both practice and actual tests, explaining that this would help them to relax. Many children are interested in animals so they were a theme of quite a few activities.
I also applied psychological research showing that processing words to do with being a professor improves your intellectual performance. We went around the room generating words to do with being a professor, such as attention to detail, enjoying a challenge, creative problem solving, precision, openness to new ideas, resilience, logical working out. This was done during a lesson and again just before pupils started the tests.

I had a poster up welcoming them to a conference on Advanced Mathematics and addressed the pupils all as Professor ……. using their surnames, and we improvised dialogues between eccentric academics when solving problems:

“Fascinating, Professor. I wonder if you have considered using one of your brand new cutting edge circle formulas to solve this intriguing problem?”

“Ah, yes, you refer of course to the infamous area of a circle = πr². An excellent suggestion!”

The rationale behind this was that acting and speaking a part would enhance their self-image as good mathematicians, and also refine the logical processes of problem solving.

Definitions and methods were attributed to individuals. For example, if Matthew found it hard to remember what the median was, it would be called “Matthew’s median” and Matthew would frequently be asked to define it, as the conference expert. This draws on psychological principles of memory: we remember concepts that are linked to something personal, memorable, humorous or emotional.

There were two main principles of showing working that children learnt to use. The first was to turn any information in the question into an equation. Examples include “distance = 10 km”; “x + x + 80° = 180°”; “20% = 42”.

The second was to use arrows on both sides of the equation to multiply or divide:

Walking around while pupils were completing the tests, or Conference Challenges for Advanced Mathematicians, as I called them, I was delighted to see numerous examples of them showing both of these methods for questions across almost all topic areas. Other successful outcomes from the ten weeks included increased confidence in approaching challenging problems and asking deep mathematical questions. Some pupils achieved a level 6, but perhaps more importantly they all exceeded their level 5 targets, for example from a 5c to a 5b, and reported a better understanding of a number of level 5 topics, such as fractions, decimals, percentages, ratio, averages, and graphs.

Seeing the pupils so relaxed and happy during the tests, and afterwards when we ate a cake they had made with πr² iced on the top, gives me hope that testing in the right environment can be a positive experience.

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Note: Powerpoint slides for the lessons described are available at www.atm.org.uk/mt241

Reference