Bob Burn comments on Jenny Murray’s article in MT234

Just a little P.S. to Jenny Murray’s piece of autobiography (MT 234, p.22):

Jenny had a disappointment when looking at the difference between consecutive cubes. The disappointment was arithmetical, but looking at \((a + 1)^3 - a^3\) geometrically, in terms of one cube inside another, explains each of the parts of the difference. Take a Dienes cube and add on one more layer: 3 blocks, 3 longs and a unit.

Seeing square numbers made up from a sum of odd numbers had delighted Jenny. This idea works for cubes in a different way:

\[1^3 = 1\]
\[2^3 = 3 + 5\]
\[3^3 = 7 + 9 + 11\]
\[4^3 = 13 + 15 + 17 + 19\], and so on.

A sum of cubes emerges from looking at triangular numbers.

\[T(2) = 1 + 2,\ T(3) = 1 + 2 + 3,\]
\[T(4) = 1 + 2 + 3 + 4.\]

A pebble diagram shows \(T(3) + T(2) = 3^2\); obviously \(T(3) - T(2) = 3\).

Multiplying these two equations together gives \(T(3)^2 - T(2)^2 = 3^3\).

If we make a list of similar equations:

\[T(4)^2 - T(3)^2 = 4^3\]
\[T(3)^2 - T(2)^2 = 3^3\]
\[T(2)^2 - T(1)^2 = 2^3\]

and add them up we get

\[T(4)^2 = 1^3 + 2^3 + 3^3 + 4^3;\]

and the pattern goes on.

Bob Burn
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