Readers delighted by the simplicity of the method of multiplication shown in MT203 (Foster, 2007) may be interested in a Gattegno-inspired variant.

Each set of intersections in the representation Colin Foster describes corresponds to a ‘window’ pane in the method known by that or similar names (‘lattice’/‘gelosia’/…). This may be as old as place value itself, and, if so, goes back to the Ancient Hindus. In ‘line multiplication’ each individual product is modelled as a rectangle of points, whereas in the lattice method, as it has come down to us via the Arabs and later European texts, each product is shown as a written number. However, in both cases the position of the number in the ‘lattice’ dictates its place value. What Gattegno does is to use the multiplication square as a ‘ready reckoner’ and code the place value of each individual product by its colour. The colour arises from the overlap of tinted acetates – the process of colour subtraction (using that word to mean the physical, not the mathematical, operation, of course). The picture to the right shows the Cuisenaire Product Finder set to multiply 973 by 682.

Paul Stephenson is operations director of The Magic Mathworks Travelling Circus.
www.magicmathworks.org

Reference

Notes
2 The Product Finder was issued by The Cuisenaire Company of America, Inc, in 1983. It is no longer in their catalogue but an internet search may turn up second-hand copies. An alternative is for you – or, better still, your pupils – to make them. The tints need not correspond to those of the original acetates but should be chosen to produce distinct colours when overlaid.

Misunderstanding of fractions! Caroline Rickard

Having just spent an hour with a Y5 class investigating their understanding of various aspects of fractions (including ratio), I wanted to share some of my observations. Not because they necessarily illustrate particular insight, nor to offer fantastic solutions, but rather because they serve as useful reminders of what children find hard. The questions were taken from an article ‘Drawing on a theoretical model to study students’ understandings of fractions’ (Charalambous and Pitta-Pantazi, 2007) but were presented in a random order rather than grouped as in the original article. I gave the questions initially as a test, asking the children to have a go at them on their own, not worrying about bypassing any they couldn’t do. After about 20 minutes I collected these in and gave out fresh sheets of the same questions, but this time to pairs of children. I asked them to talk together about the questions they had tried and to be prepared to report back on:
• any questions answered differently and which they had been unable to resolve.

The following questions proved the most controversial! This first question was attributed to Noelting (1978; in Charalambous and Pitta-Pantazi, 2007) and states:

John and Mary are preparing orange juice for their party. Presented below are the recipes they used. What recipe will make the juice most ‘orangey’?

John’s recipe: Two cups of concentrate juice – five cups of water
Mary’s recipe: Four cups of concentrate juice – eight cups of water.

The two girls who chose to talk about this question chose it because they were sure they were right, and indeed they were, giving some sound reasoning.
relating to Mary using twice as much water as juice and John’s recipe using more than twice as much and therefore being weaker. Those who failed to appreciate this argument were convinced that because Mary was using more water, hers had to be more watery and less ‘orangey’. Now, does this say something about their lack of understanding of the mathematics of ratio, or could we attribute the difficulties to lack of appreciation of the ‘real-life’ scenario? I’ll leave you to draw your own conclusions.

The second question which caused differences of opinion was: 

\[ 2 \div \frac{1}{2} = \]

and the two girls who offered this question for discussion did so because one believed the answer to be 1 and the other claimed it was 4.

They both gave nice explanations – about how 2 halved was 1 and the other girl spoke of the 4 halves in 2 when shared out. The children voted as they lined up for assembly: answer ‘1’ was by far the more popular…!

Last but not least, a comment from a child who was disappointed to find that his classmates didn’t agree with his choice of this diagram to represent 2/3. “Well, the question doesn’t say the pieces have to be equal!”

The misunderstandings apparent here will come as no surprise to many experienced practitioners, I’m sure. But the paired discussion provided a wonderful opportunity to explore the children’s views in a non-threatening manner, and hopefully to begin to address some of their misconceptions.

Caroline Rickard works with teachers in south coast schools.


---

**Line multiplication – a response to Colin Foster (MT203)**

Marion Walter

First, I kicked myself! I often problem pose on a situation – and I certainly have sometimes thought of, say, \( 3 \times 4 \) using lines and their intersections, so how come I never have asked myself if I had the product of 2 – or more – digit numbers could one find the product by line multiplication, other than by unmanageable counting of the intersection of an unwieldy drawing of, say, 43 and 21 lines! I really liked the piece, but I think it is not quite as simple as described.

I think that stating that one draws the lines for the second number ‘going the other way’ is not sufficient. I agree about drawing the first set of four parallel lines (call them A), representing the 40, and the second set of three lines (call them B) representing the 3, parallel to the first set. But then when you draw the two parallel lines (call them C) representing the 20 it matters how far from C you draw the next set of lines (call them D – in this case only one line) parallel to C. The distance between the lines A and B does not have to be the same as the distance between the lines C and D, but somehow you have to get the intersection of A and D to be vertically above the intersection of C and B (see Figure 1).

You can construct D like this, but it is not necessary. The vertical line up is actually an artificial condition, because one can turn the paper if partial products to be added are not on a vertical line. If one draws D parallel to C at a random distance, one can just turn the paper to get the 10s places to line up vertically (see Figure 2).

However, for products involving 3-digit numbers, giving rise to, say, three sets of parallel lines (A, B, C and D, E, F), spaces between A, B and C should be equal, as should also be the spaces between D, E and F, to make the partial products line up. To get the partial products to line up vertically, one can repeatedly use the arrow procedure shown in Figure 1.

Marion Walter taught for many years at the University of Oregon, Eugene, Oregon, USA, but is now retired. She misses coming to the ATM meetings.

walter@math.uoregon.edu
The attached document has been downloaded or otherwise acquired from the website of the Association of Teachers of Mathematics (ATM) at www.atm.org.uk.

Legitimate uses of this document include printing of one copy for personal use, reasonable duplication for academic and educational purposes. It may not be used for any other purpose in any way that may be deleterious to the work, aims, principles or ends of ATM.

Neither the original electronic or digital version nor this paper version, no matter by whom or in what form it is reproduced, may be re-published, transmitted electronically or digitally, projected or otherwise used outside the above standard copyright permissions. The electronic or digital version may not be uploaded to a website or other server. In addition to the evident watermark the files are digitally watermarked such that they can be found on the Internet wherever they may be posted.

Any copies of this document MUST be accompanied by a copy of this page in its entirety.

If you want to reproduce this document beyond the restricted permissions here, then application MUST be made for EXPRESS permission to copyright@atm.org.uk

---

**Membership of the ATM will help you through**

- Six issues per year of a professional journal, which focus on the learning and teaching of maths. Ideas for the classroom, personal experiences and shared thoughts about developing learners’ understanding.
- Professional development courses tailored to your needs. Agree the content with us and we do the rest.
- Easter conference, which brings together teachers interested in learning and teaching mathematics, with excellent speakers and workshops and seminars led by experienced facilitators.
- Regular e-newsletters keeping you up to date with developments in the learning and teaching of mathematics.
- Generous discounts on a wide range of publications and software.
- A network of mathematics educators around the United Kingdom to share good practice or ask advice.
- Active campaigning. The ATM campaigns at all levels towards: encouraging increased understanding and enjoyment of mathematics; encouraging increased understanding of how people learn mathematics; encouraging the sharing and evaluation of teaching and learning strategies and practices; promoting the exploration of new ideas and possibilities and initiating and contributing to discussion of and developments in mathematics education at all levels.
- Representation on national bodies helping to formulate policy in mathematics education.
- Software demonstrations by arrangement.

**Personal members get the following additional benefits:**

- Access to a members only part of the popular ATM website giving you access to sample materials and up to date information.
- Advice on resources, curriculum development and current research relating to mathematics education.
- Optional membership of a working group being inspired by working with other colleagues on a specific project.
- Special rates at the annual conference
- Information about current legislation relating to your job.
- Tax deductible personal subscription, making it even better value

**Additional benefits**

The ATM is constantly looking to improve the benefits for members. Please visit www.atm.org.uk regularly for new details.

**LINK:** www.atm.org.uk/join/index.html