GOING ROUND IN CIRCLES

Tandi Clausen-May describes some Powerpoint presentations designed to help visual learners calculate the circumference and the area of a circle.

Muddled formulae

Our school curriculum is still very largely print-based, and visual and kinesthetic learners may have difficulty accessing it. The recitation and the symbols involved in counting and recording with numbers and the routine procedures of calculation are hard to grasp and easy to forget. With the advent of the numeracy strategy, mental images – number lines or the area multiplication method, for example – have become far more common in our classrooms. These can help many of our pupils to understand and recall the concepts, not just the routines, of numerical calculation (Clausen-May, 2005).

But what about shape and space? Surely here we can expect visual and kinesthetic learners to shine! Unfortunately, this is not necessarily the case. It depends how the concepts and principles are presented. Take the formulae for the circumference and area of a circle. These are constantly muddled by pupils who have been taught them – or, at least, have tried to learn them – by rote. Is the area of a circle $2\pi r^2$? Or $\pi r^2$? Or $2\pi r$? The confusion is endless! If circumference and area are taught as formulae then, for many pupils, they will be meaningless – and eminently forgettable.

But even a more practical approach may fail to get the concepts across. The key concept here is $\pi$. We have all wrapped a bit of string around a Coke can or a tin of baked beans and then demonstrated triumphantly that the same bit of string will stretch across the top three or so times. This is a useful activity, certainly – but it is quite time-consuming, so it is not likely to be repeated more than once or twice. Then we are back with the ‘rule’, which pupils may apply with little understanding.

What many pupils need is a mental image, a dynamic model on which they can hang their understanding of the crucial concept of $\pi$. Then they can recall not the value but the image from which the value may be derived. The capacity of Powerpoint to show movement is useful here. This article describes how a pair of Powerpoint presentations can be used to establish mental images that can help pupils to understand, and so recall, the principles that underlie the formulae.

Powerpoint pi

$\pi$ is introduced with a dynamic image which is designed to convey the relationship between the circumference and the diameter of the circle. It is defined as ‘the number of times you must travel straight across the circle to go the same distance as all the way round the circle’. This is shown by the movement of two dots, one travelling around the circumference and the other across the diameter (figure 1).

The next screen asks, ‘How can we be sure that $\pi$ is a bit more than 3?’ The program demonstrates that the ‘three and a bit’ is not just a trick by comparing the distance around a circle with the distance around a regular hexagon that just fits inside it. One dot travels back and forth across the hexagon while the other travels around it (figure 2).

Since the hexagon is made up of equilateral triangles, the distance straight
across the middle (the diameter) is exactly one third of the distance round the edge – so we can see that ‘the distance all the way round is exactly 3 times the distance straight across the middle’. Then the distance round the hexagon is compared to the distance around the circle that just contains it – which is clearly slightly greater, so $\pi$ is a little bit more than 3 (figure 3).

### The area of a triangle circle

Having established that ‘circumference $= \pi \times$ diameter’, where $\pi$ is a little bit more than 3, the next presentation goes on to build up an image for the formula for the area of a circle. Here, the idea of opening the circle out into a triangle is used (figure 4).

The dimensions of the circle are transferred onto the triangle and the triangle is sheared into half a rectangle (figure 5).

The formula to which this leads, $\text{Area} = \frac{\text{Circumference} \times \text{Radius}}{2}$, is not the standard one found on the formula sheet. However, it is equivalent and it makes much better sense. To work out the area of a circle you start by finding its circumference – a bit more than three times the distance straight across, or $\pi$ times the diameter’. Then, recalling the image of the circle opening out into a triangle, it becomes clear why you must multiply the circumference by the radius and halve the result. The formula for the area of a circle is exactly the same as the formula for the area of a triangle – it is just expressed in terms of circumference and radius, rather than base and height.
Some pupils (and some teachers!) may feel more comfortable when they have manipulated the algebra to prove that this really is equivalent to the standard formula \( A = \pi r^2 \). But for others, the image that links the circle directly to the triangle, and thus to the same formula for the area, is much more convincing – and therefore more memorable.

The two Powerpoint presentations described here may be downloaded from www.atm.org.uk/free-resources/index.html. I would be very interested to hear how teachers and pupils use the presentations; for example, to introduce or to revise the formulae for the circumference and area of a circle.

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Note

1 To download these Powerpoint files, go to www.atm.org.uk/free-resources/index.html. Please note that they work only on versions of Powerpoint from 2003.

Reference