**RESPONDING TO GEOFF FAUX’S CHALLENGE (MT189 p10)**

Rotate the blue triangle through 90° about its common vertex with the grey triangle. The resulting big (blue and grey) triangle is divided in two by its median and hence the two pieces are equal in area.

By the way, when the diagonals of a parallelogram are drawn the parallelogram is divided into four triangles of equal area – two congruent pairs. The grey triangle is such a quarter of three different parallelograms. The ‘other’ triangles in these three parallelograms are the three triangles which are to be proved equal in area to the grey triangle.

_Derek Ball_

In _MT189_, Geoff Faux challenges us to find a different proof of Cross’s Theorem, which states that in the geometric configuration in _figure 1_, where squares are drawn on the three sides of a general triangle, the four triangles all have equal areas.

Our PGCE group considered this question, looking for proofs of Cross’s Theorem. In order to explore the properties of this configuration, we experimented (and even played) with _Cabri_ and _Geometer’s Sketchpad_ in order to get a feeling for the situation. (_Geometer’s Sketchpad_ allows for measuring areas, which was very useful for our investigations.) Along the way, the sketches suggested some interesting plausible conjectures; however, one of the superb features of using dynamic geometry software is that it is easy to adjust the original triangle to test such conjectures empirically, and some of the conjectures were thus quickly consigned to the ‘failed attempts’ basket.

We succeeded in producing the following proof. Consider _figure 2_. We aim to show that the two shaded triangles, _ABC_ and _PBQ_ have the same area; the result then follows immediately. We have drawn in extensions of the segments _AB_ and _PB_, and constructed parallels to them through _P_ and _Q_ respectively. We label the intersection of _AB_ extended with the parallel through _Q_ as _X_. We also dropped a perpendicular to _PB_ extended from vertex _C_, meeting _PB_ extended at _Y_. Now, the area of _PBQ_, half base times height, is given by \( \frac{1}{2} PB \cdot BX \), and the area of _ABC_ is \( \frac{1}{2} AB \cdot BY \). (Note that _BY_ is perpendicular to _AB_, since \( \angle PBA \) is a right angle.) But _BC_ = _BQ_ (sides of a square); \( \angle BXQ \) and \( \angle BYC \) are both right angles, and \( \angle XBQ \) and \( \angle YBC \) are easily seen to be equal. Thus _BXQ_ and _BYC_ are congruent, so _BX_ = _BY_. Since _PB_ = _AB_, it follows that the two triangles _PBQ_ and _ABC_ have equal area.

Although this argument might seem to rely on the angle _ABC_ being acute, it is important to check that the results do not depend upon the actual configuration. (This is often missed in textbooks proving the result that the angle at the centre of a circle is twice that at the circumference: they only consider one possible configuration.) A quick look at _figure 3_, however, shows that the argument follows identically if the angle is obtuse. The situation is trivial if _ABC_ is a right angle.

Paul Andrews, our mathematics lecturer, then suggested extending the result to quadrilaterals, pentagons and even possibly beyond. Our investigations then led to an interesting result for quadrilaterals, but failed to reveal anything for pentagons or beyond.

The benefits of using dynamic geometry software for visualising geometric scenarios became very clear to us. Their incredible ability for also showing which features of a problem are essential and which are coincidental also helped us to comprehend the essence of the situation.

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