This article was written in response to an article by Paula McLoughlin printed in Dec Mt185 entitled "L'originalitat consisteix a retornar a l'origen".

You can read the original article here: http://www.atm.org.uk/journals/mathematicsteaching/mt185files/loriginalitat.pdf

CURVE STITCHING WITH LOGO
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The article by Paula McLoughlin [1] describes how some familiar curve-stitching patterns can be drawn using Logo. The ideas in that article can be developed in a variety of ways, but here we consider the Logo ideas involved, and some ways of using Logo so that the drawings can be developed with much less labour.

There are easy ways into Logo, and at every level the commands are powerful. But it seems a mistake to restrict oneself to a few particular commands when the introduction of a few more can make operations much easier. Drawing the curve stitching patterns based on the equilateral triangle, indeed drawing the patterns on any triangle, requires no trigonometry at all, and the trigonometry involved in drawing epicycloids requires no more than a knowledge of what the sine and cosine are. The important thing is not to be tied to the basic four operations (FD, BK, LT, RT,) of the turtle. Some Logo users appear to think that it is unsporting to use anything else, but turtles are capable of much more and restricting the use of the turtle to these four moves is like going round the golf course with one club.

Turtles are usually introduced by the four moves described; but turtles adapt very easily to Cartesian coordinates. The computer screen is, in effect, a piece of graph paper, although the grid is invisible. We can move the turtle to different positions by setting the coordinates of the position we want. Thus SETXY 0 0 moves the turtle to the origin; SETXY 200 100 moves the turtle to the point with coordinates (200,100), and so on. Try various inputs and see the layout of the screen. When this idea is used with the commands PU (PENUP) and PD (PENDOWN) we have a simple, but powerful, drawing device; we can move to some new point without leaving a mark or we can trace in a line segment as we go. We have in fact the system that was available on some of the earliest microcomputers to come into schools. The BBC computer had the commands MOVE and DRAW.

The command SETXY enables us to define a procedure to draw the line segment connecting the points (X1,Y1) and (X2,Y2). (We use symbols which can be typed in Logo, rather than the symbols more commonly used in mathematics books.)

TO SEG :X1 :Y1 :X2 :Y2
  PU SETXY :X1 :Y1
  PD SEYXY :X2 :Y2 PU
END

We lift the pen up; take it to (X1,Y1); put it down on the paper; move it to (X2,Y2); and then lift the pen up again (this final move may not be needed, but it is a precaution with an eye to the future).

To draw the configuration which is the basis of most curve-stitching we have to draw segments joining points on two lines which are the arms of an angle, meeting at a vertex. One of the points may step along the y-axis (for example) and the other may step along the x-axis. We will define a procedure which takes one point from (0,0) along the y-axis to (0,200), in steps of 20, and the other from (200,0) along the x-axis to (0,0) in steps of 20. As we go we will join the points up using the SEG command. (See Fig 1.)

When a computer has to repeat an action a number of times various methods are possible. Since the earliest days of Logo it has been customary to employ recursion. This is explained in almost any elementary introduction to the language – it is Logo’s way of incorporating the idea of “and so on”,
which enables us to hand over repetitive tasks to the machine instead of having to give separate
instructions each time.

It seems that some people have difficulty in coming to terms with recursion, and this holds up their
progress with the language. But modern versions of Logo contain more options than did the versions
of the language which were in use (say) twenty years ago. MSW Logo contains a FOR instruction,
which enables you to use commands which in another language might be expressed as “FOR X=1 TO
10 . . . .”. MSW Logo also includes the word “REPCOUNT”, which you can use with REPEAT to
build up for-loops in which the counter is of the simplest form. These commands are explained in the
help files that download with the language, and they provide alternatives to recursion. But we will
use recursion, so that our procedures are easily adapted to other versions of Logo which may not
contain these innovations.

The essential ideas are so simple that people are surprised that there is anything to gain by spelling
them out in detail. But machines have to be given instructions in punctilious detail.

Imagine yourself constructing a stitching pattern by drawing segments. At any particular time you
have drawn some segments, and the REMaining SEGments are still to do. How do you draw the
REMSEGS? You draw the first one, and then draw the ones which remain after that. And, since
computers take nothing for granted you have to be sure to tell the computer that when no more
segments remain it must STOP! It is amazing that this kind of thinking is so powerful! And I believe
that the failure of some people to take it seriously has held up their progress in the language in the past.

This leads to the procedure,

```
TO REMSEGS :A :B :C :D :M
SEG :A :B :C :D
IF :M=0 [STOP]
REMSEGS :A :B + 20 :C – 20 :D :M – 1
END
```

The variable M is a counter – it counts how many segments are still to be drawn. At the start we input
the number of segments which we wish to draw, and this number diminishes by one each time round.
When M becomes 0 the process stops. Observe that in our case we wish to move the ends of the
segment by 20 each time, and that we will draw 11 segments in all. Note why the stop instruction is
where it is. We want to draw “segment 0” before we stop. You may try putting the stop instruction in
other places in order to see what happens.

Now if we type in the command REMSEGS 0 0 200 0 10 we start off with the correct values and the
machine draws segments until the stitching is complete.

![Fig 1](image)

This procedure draws the stitches inside a particular right-angled triangle, we will later see how to
adapt this to draw inside any triangle.
The method of using Cartesian coordinates with SETXY, rather than turtle movements, has some disadvantages; for example we cannot produce extensions of a pattern by simple saying RT90 (for example) and then drawing again from the fresh starting position. We usually have to decide ourselves where the fresh starting points have to be so that we know where to SETXY. But in many circumstances these things can be easily worked out – or failing that we may get the machine to work them out for us.

We would like to be able to draw the above pattern of stitches inside any assigned triangle, and we will show later how to do this - without any trigonometry – but for the moment let us give another simple example using SETXY. We will draw some of the attractive curve-stitching patterns which are based on circles – the epicycloids. These do not require any trigonometry in the sense of the mensuration of triangles. We only need to know what is perhaps the simplest property of SINE and COSINE. We need the idea of the unit circle, and the idea of locating points on it by means of their associated angles. The cosine and sine of a given angle are simply the x and y coordinates of the associated point. (This may be taken as the definition of cosine and sine, and indeed some school texts have done things this way.) To get the coordinates of a point on a circle of radius R we simply have to scale up these values. We do not need to look up sines and cosines in tables, because Logo can look them up for us.

Epicycloid patterns can be constructed by stitching threads between points on a circle. In the simplest case we join the point whose angle is 10° (say) to the point whose angle is twice this; in this case the point whose angle is 20°. In turn the point with angle 20° is joined to the point with angle 40°, and so on. We will now see how to do this with Logo. It will be convenient to work with a circle of radius 200.

First we need the procedure to draw a thread starting at the point associated with some angle, and terminating at the point with twice the angle.

```
TO THREAD :ANG
PU SETXY 200*COS(:ANG) 200*SIN(:ANG)
PD SETXY 200*COS(2*:ANG) 200*SIN(2*:ANG) PU
END
```

Now, just as before, we need a recursive procedure to draw the ‘remaining threads’.

```
TO REMTHREADS :N :ANG :INC
THREAD :ANG
IF :N=0 [STOP]
REMTHREADS :N-1 (:ANG-:INC) :INC
END
```

The brackets in line 4 of this procedure are not required by the machine, but they make it easier for us to read; and make it easier to check that we are indeed providing the right number of variables for the recursive call we are making to REMTHREADS.

It is useful now to define a procedure which calls REMTHREADS with suitable input values; this will enable us to experiment by drawing the diagram with different numbers of tangent lines;

```
TO EPI :N
REMTHREADS :N 360 –360/:N
END
```

Now the command EPI 100 draws the figure with 100 threads. (Fig 2).

Diagrams of this kind have applications to shuffling a pack of cards and to explaining the shape of the curve of light which we sometimes see in the top of a cup of tea. The method of construction which we have employed gives clues as to why this is.

Try altering the multiplier from 2 to 3 or 4, and see how this affects the result.
Today’s computers are very much faster than computers were twenty years ago; and this is not always an advantage. Then you watched the turtle’s ponderous movements, and if you had given an incorrect instruction you could immediately see where things went wrong. Today the fun of seeing a turtle trace out a drawing is largely lost, because drawings such as we are producing here appear in a flash. But something can be done about this by using the sound command to cause delays. If you include a command such as SOUND [400 1000] somewhere in the drawing loop, perhaps after drawing a THREAD, the machine will wait while it outputs the sound and you have the pleasure watching the threads jump in one at a time.

We next want to do curve-stitching constructions similar to those in Fig 1 inside lines of more general position. We have to handle any given coordinates for the vertex (VX, VY) and the ends of the two arms (AX, AY) and (BX, BY). We also provide a parameter N for the total number of threads to be drawn, and a counter M so that the machine can keep a check on how far it has got. There are often advantages in counting down rather than counting up.

One might think that this requires extensive trigonometry, and indeed it does if one is doing the construction with the limited range of turtle movements FD, BK, RT, LT; but all that is needed is the formula in coordinate geometry giving the coordinates of the point dividing a line segment in a given ratio. The coordinates of the point dividing the line joining (X1, Y1) and (X2, Y2) in the ratio p:q are

\[ X = \frac{pX2 + qX1}{p + q} \quad \text{and} \quad Y = \frac{pY2 + qY1}{p + q}. \]

In ordinary algebra we denote multiplication by juxtaposition, but since we intend to use these formulae in Logo, where multiplication is denoted by an asterisk, we use an asterisk here.

To use the above formulae in Logo we can define the procedure

```plaintext
TO PTDIV :st :fin :p :q
  OP (:p*:fin + :q*:st)/(p + q)
END
```

Fig 2
The command OP ‘outputs’ the result of a calculation. It is usually needed in Logo when the result of a calculation is handed on to be used elsewhere. We will need to use this procedure on the x and y coordinates of the points at the start and finish of the segments which we want to draw.

We will extend the method used before; but instead of specifying a definite number of steps we introduce an algebraic variable so that a given line segment is sub-divided by a number of stitches, N, which can be decided later. We also need a counter, M, to halt the process when the right number of segments have been drawn.

The base points are to be specified initially, and we have to locate the ends of the thread at its various positions. The procedure SEG can be just as before, but we need to redefine REMSEGS to accommodate the larger number of variables, and to include the appropriate calls to the procedure PTDIV. So we get

```
IF :M=0 [STOP]
END
```

The brackets in the second line help us to check that we are making the correct inputs to the procedure SEG.

This completes the essential procedures, but it helps to have a procedure to call REMSEGS2 with the appropriate values; so we define
```
END
```

To illustrate the use of this we give Fig 3, which uses this procedure three times, setting the pen in different colours and taking the three points (150,0), (-50,125), (-75,-150) as the vertex (VX,VY) in turn, with the other two as the ends of the arms, and drawing 21 threads in each case.

![Fig 3](image)

REFERENCES
1. Paula McLoughlin: “L’originalitat consisteix a retornar a l’origen” as Gaudi would say.
   
   The following free downloads give many ideas for using Logo, including examples of recursion and references to further sources:
   2. www.softronix.com/logo.html

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