1–100 RULES OK?

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In a recent edition of *Mathematics Teaching* Midge Pasternack [1] argued the case for the use of the 0–99 square with young children rather than the ubiquitous 1–100 square. I would like to take this opportunity to mount a defence in favour of the much maligned 1–100 square.

My main criticism of the 0-99 square (apart from the linguistic and logical difficulty of having a 100 square that only goes as far as 99) is that it does not support the development of a particularly important concept in number understanding for young learners, namely, the relationship between cardinal and ordinal number. If children are going to make sense of number lines to the extent that they become confident with empty number lines they need to understand this relationship. I feel very strongly that the 0-99 square actually prevents children from making this important connection.

**Counting**

There is almost universal agreement that children’s understanding of number develops from and builds upon their ability to count, and by ‘count’ I do not mean simply being able to recite the number names – although knowing these by heart and in the correct order is an essential part of being able to count. (If you only know the first six number names, you will have great difficulty counting seven objects).

The ability to count also involves the matching of these number names in one-one correspondence with the objects to be counted, ie, ensuring that the same number name is not matched to two different objects and that each object to be counted has one, and only one, number name assigned to it. Children must also appreciate that the number name assigned to the final object in a count – no matter in what order the objects have been counted – gives the numerosity or size of the collection. This last requirement involves an appreciation of the cardinal principle.

**Cardinality**

When children are first introduced to the 1–10 number strip and later to the 100-square we need to ensure that cardinality is maintained, ie, the number in the last square should represent the total number of squares in the model. So, in a 1–100 square the number 37 will be in the 37th square, and consequently 37 can be seen to represent 3 ten strips and 7 extra squares. It could be argued that the 100-square should actually be introduced by extending the 1–10 strip, cutting this extension into separate sections containing ten squares and repositioning them underneath each other.

A similar development can be found in the Dutch approach to the number line (they dropped the 100-square model just at the time when the national numeracy strategy adopted it in a big way!). In Holland great use is made of the 100-bead string (now slowly making an appearance in this country) both for developing general number sense and for facilitating calculation. When the time comes to introduce the number line, as part of an eventual aim of getting children to work with an empty number line, great care is taken in the early stages to ensure that the lines are always accompanied by diagrams of bead strings of different lengths. A similar approach to introducing number lines can be found in the national numeracy strategy early years progression charts and individual teaching programmes. [2]

**Counting difficulties**

Research findings on the counting difficulties experienced by young children have been described and discussed in various journals [3, 4]. One such difficulty involves articulating the progression from, say, 19 to 20, or from any number ending in nine to the next decade number (even though it is actually the same decade!). The 1–100 square situates multiples of ten on the same row as all the other numbers in the same decade and adjacent to the number ending in nine. This helps children to make the link smoothly when counting, whereas the 0–99 square exaggerates the problem by having the decade numbers at the beginning of the following row. The argument given in the original article against having 100 on the same grid is that it is a different power of ten. I would suggest that having the 100 reinforces an important counting link between 99 and 100.
Place value

The argument in the original article is that a major strength of the 0–99 chart lies in the fact that it groups in one row all the numbers that have the same number of tens. But surely, this is a less powerful argument than the one made above, namely, that the 1–100 square shows all two-digit numbers as comprising the number of rows denoted by the first digit plus the number of individual squares denoted by the second. This useful model of the structure of two-digit numbers does not work on the 0–99 square. From the perspective of place value, another argument for having 100 on the grid is that considering the ten rows both separately and together reinforces the fact that 10 tens make 100.

Patterns

One section of the article considers a range of patterns on the 0–99 square. Whilst accepting the way in which the vertical line of symmetry on the 0–99 square provides a useful mnemonic for rounding (0–4 on one side and 5–9 on the other), one could argue that the horizontal line of symmetry doesn’t appear to divide the square in half (49 compared with 99!). And one only has to look at the 100th edition of Mathematics Teaching (One hundred things to do with a hundred square, [5]) to appreciate the myriad activities involving patterns using such a square. To my knowledge there is no extant set of similarly useful activities for the 0–99 square!

Calculation

One of the most important and generalisable basic mental calculation strategies (despite the fact that it is not awarded key objective status by the national numeracy strategy) is bridging, either through ten or through a multiple of ten. This strategy includes bridging up for addition (17 + 8 calculated as 17 + 3 = 20; 20 + 5 = 25), and bridging down for subtraction (23 – 7 calculated as 23 – 3 = 20; 20 – 4 = 16). It uses knowledge of complements in ten and the ability to partition one-digit numbers in every possible way. On the 1–100 square a calculation such as 36 + 7 can be taught as ’jump to the end of the row (from 36 to 40), and then jump the remaining 3 to 43’. It is much more awkward and less logical for children to use the same strategy on the 0–99 square.

One of the national numeracy strategy Y4 objectives is ’Derive quickly all number pairs that total 100’, and one of the models suggested for the development of these ’complements in 100’ is based on a blank 100-square. For this to make sense to children the square would have to be thought of as a 1–100 rather than a 0–99 square. An argument in support of this latter format is that 16 – 16 can be solved on the 0–99 (jump back 16 squares from 16 and you land on zero) but not the 1–100 square. I would argue that, given the cardinal/ordinal argument above, if there are 16 squares and you take 16 away (this can be done practically) there are obviously none left.

Concluding thoughts

Writing this article has actually made me feel less averse to the 0–99 square than I was before I began it. I still believe very strongly that the 1–100 square should be introduced and worked on first as a ’cardinal model’, but now feel that it should be followed later by the 0–99 square in an ’ordinal model’ that introduces zero and parallels work on the number line. The appearance of the multiples of 9 and 11 on the two diagonals makes the 0–99 square more appropriate for older children learning their multiplication facts.

To complete the move from counting tangible objects to shifting backwards and forwards along a number line I would like to tentatively suggest the following progression, in which several of the later ‘stages’ would overlap:

- Counting
- 1–10 number strip
- 1–100 square
- 0–99 square
- Number line
- Empty number line.

So, returning to the title of the original article 0–99 or 1–100? the answer is… both, provided that 1–100 comes first!

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References

1 M. Pasternack: ’0-99 or 1-100?’, Mathematics Teaching 182, ATM, March 2003, pp. 34-35.
2 National Numeracy Strategy progression charts: Using models and images to support mathematics teaching and learning in Years 1 to 3. See information available at: http://www.standards.dfes.gov.uk/numeracy/about/?a=whole_article&art_id=10797
5 Mathematics Teaching 100, ‘One hundred things to do with a hundred square’, ATM, September 1982, pp 38–41
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