At Key Stage 3, students are expected to learn about 2-dimensional shapes and their properties, including equal sides and angles, parallel and perpendicular sides, and symmetry. One only needs to look at textbook schemes to see that this can lead to a very dry activity when students are expected to learn the names of shapes and their properties, or recognise shapes, from a list of properties. Yet the Framework for Teaching Mathematics at Key Stage Three says: ‘pupils should be taught to identify and use the geometric properties of triangles, quadrilaterals and other polygons to solve problems; explain and justify inferences and deductions using mathematical reasoning’ (DfES, 2001). Consequently we need to design activities, which encourage students to think more deeply and gain greater insight into shapes and their properties. The task described here aims to address these issues.

The task
Imagine a toy kite, such as illustrated in figure 1 - which could be flown on a windy day and which has the same shape as a mathematical kite. The structure of this kite is based on two perpendicular sticks, or bars, which have unequal lengths and which are at right angles to each other. Usually, one bar is the perpendicular bisector of the other and this gives the kite, symmetry in one axis. The bars are covered with some kind of textile which is stretched over them to make the kite shape. Now, if the textile was stretchable and the bars could be moved side-to-side, or even rotated, many different mathematical shapes could be created. This would be very difficult to produce in the real world, but it can be done in the human imagination, and on the computer in a dynamic geometry environment.

Thus the computer offers a way of working that helps students to access approaches and solutions that would not be available to them using pencil and paper (Hoyles and Noss, 1992).

The task files were made in the Geometer’s Sketchpad, version 4, (Jackiw, 2001). The first file contains an 8cm vertical bar and a 6cm horizontal bar, which have been constructed so that the lengths and orientation remain constant. The bars are constructed separately and then placed one over the other. The ends of the bars are joined using the line tool to create a quadrilateral. The interior of the quadrilateral is constructed which fills the shape with colour. Once the shape is complete the bars can be dragged inside the shape generating particular triangles and quadrilaterals.

Figure 2 shows possible shapes generated by dragging the bars into different positions. The bars are shown, by the thicker lines on the figures. The bars - represented by the lines AC and BD - as in Figure 3 - are actually the diagonals of the quadrilateral, or base and height of the triangle. When dragged inside the shape the bars generate right angled triangles, isosceles triangles, a rhombus, various kites and arrowhead kites as well as irregular triangles and quadrilaterals. An interesting question to consider is ‘Where do the bars cross each other?’ to make any specific shape.
LEARNING ABOUT PROPERTIES OF 2-D SHAPES FROM THE INSIDE OUT

The question of where the bars cross is akin to enquiring about the properties of the diagonals in a quadrilateral, or the properties of the base and height of a triangle. Thus students have the opportunity to observe that the diagonals of a kite are perpendicular and one of them bisects the other, the height of an isosceles triangle bisects the base and so on. In my experience students use more informal language to describe this - for example, in Fig. 3 the bar AC crosses the bar BD in the middle - but this can provide an opportunity for students to develop correct use of mathematical vocabulary.

First lesson
What follows is an account of a year 7 class working with the computer files for two lessons. Students used the files made in the Geometer’s Sketchpad software to construct special quadrilaterals drawing on their knowledge of the properties of such quadrilaterals. We were keen to provide learners with opportunities to discuss results and opinions and were fortunate to have three adults working with 15 pairs of students - Dave their regular class teacher, Sue who is researching into geometry tasks using Dynamic Geometry Software, and Bhavini who was a PGCE student at the time.

During the previous six months, the students had considered the properties of special quadrilaterals when completing drawings on dotted paper. At the beginning of the computer session students reviewed this work during the lesson starter, which challenged them to give the correct mathematical name to a selection of cardboard shapes.

The software was new to the students, but the task was designed so that they were able to make a start without having to learn how to use the software. Students were first provided with a file containing a shape with perpendicular bars of length 8cm and 6cm, and asked to investigate which shapes they could make by dragging the bars. This prompted plenty of discussion about which shapes they could make – although students tended to view the shapes holistically at first and not to use properties of the quadrilaterals to justify their decisions.

Students were then introduced to tools in the Geometers Sketchpad menu which they could use to measure the side-lengths and angles of shapes. They could then justify that they had made particular shapes using the angle and side properties. If the measurements which were expected to be equal were not exactly so, students were able to make small adjustments to the positions of the bars in order to get them equal, or very close to it - see Figure 3. In certain circumstances students were unable to get sides of, say, a rhombus, exactly equal - according to the displayed measurements - this provided an opportunity to discuss rounding and appropriate degrees of accuracy.

During the previous six months, the students had considered the properties of special quadrilaterals when completing drawings on dotted paper. At the beginning of the computer session students reviewed this work during the lesson starter, which challenged them to
had previously thought of a rhombus as a squashed square - see Figure 4.

On the one hand this meant that the name rhombus had been remembered, on the other hand students were puzzled that they could not turn it into a square by ‘dragging’. Towards the end of the first lesson many students realised they could not make a square using the 8cm and 6cm perpendicular bars. When asked which set of bars they would like to order for next lesson - so that they could make a square, students observed that they would need bars of equal length - they had obviously been thinking about the diagonal properties of a square. Learners also saw the connection between the rhombus and the square in that both shapes have perpendicular diagonals which cut each other in half - in student speak - or, the diagonals are the perpendicular bisectors of each other.

**Second lesson**

In the second lesson students were given a file containing two equal perpendicular bars each measuring 8cm. They were quickly able to construct a square as in Figure 5 and realised that a rectangle with non-equal sides would require non-perpendicular bars. A number of students investigated two other files, which contained bars at an adjustable angle and successfully generated the rectangle - using equal length bars, - and the parallelogram - using unequal length bars. Throughout the task, whenever students successfully generated a shape they were asked to describe the positions of the bars relative to each other. At first they found it difficult to describe the positions and so we prompted them with questions such as “where do the bars cross each other?”, “are the bars equal in length or unequal?”, and “are the bars at right angles or not?”.

Next, students were challenged to construct a square from scratch - using what they had learned about the positions of the bars inside the square. The target was for students to produce a shape that could not be altered simply by ‘dragging’ one point. Using the line tool in the software to draw a square quickly was easy - however these ‘squares’ could easily be altered by moving just one point. Students had to work out a way to construct a square, which would remain a square when it was ‘dragged’.

We provided many clues and help by showing students how to use the tools in the transformation menu. Learners then had to work out how to start with a single point and move on to construct three other points whose position depended on the first. This led to a great deal of discussion about translations and rotations - many students came close, but only a few managed to complete this challenge. The most successful method involved drawing a line, constructing its mid-point, rotating the line around the mid-point by ninety degrees, then joining the ends of the lines to complete the square. Figures 6, and 7 show how this can be done - and see over.
Once students had constructed a square they measured the sides and angles to check the properties. If the construction was successful then the sides and angles were always equal, even when objects on the square were ‘dragged’.

Whilst earlier in the year, students had considered properties of quadrilaterals in terms of drawing them on paper, this task with the computer promoted more discussion – using the correct vocabulary. The majority of students were heavily engaged in the task and were eager to discuss their results and opinions with the adults in the room. We did notice that some – high ability - learners found the task tricky in that there was not one single correct solution - they seemed to be perturbed by the open nature of the task whereas other students were very keen to try out different strategies until they found one that worked.

Discussion

The form of this task using Dynamic Geometry Software allows students to investigate shapes by dragging, and observing what happens. Using the measurements for sides and angles helped students to review and check what they already knew of the properties of the shapes. The new ideas, which students were being introduced to were the properties of the diagonals inside the shapes – or the base and height of a triangle.

We would argue that there is value in encouraging students to look at shapes, and shape properties from a different perspective - from the inside out! If learners are able to conceptualise the properties of the diagonals of a quadrilateral, and make connections to the properties of its sides and angles then students are engaging in higher order reasoning, increasing their understanding of geometry.

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References

DfES (2001) Framework for teaching mathematics at Key Stage 3


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