



The Association of Teachers of Mathematics  
for mathematics educators  
primary, secondary and higher



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7 Prime Industrial Park • Shaftesbury St • Derby • DE23 8YB • +44 (0) 1332 346599 • www.atm.org.uk • info@atm.org.uk

## **In Mathematics Teaching 196 (May 2006), ATM published its letter to the National Strategy about the revision of the Primary National Strategy.**

Read the article here: <http://www.atm.org.uk/mt/archive/mt196files/ATM-MT196-26-27.pdf>

### **The Revised Framework: a document to count on?**

*'When I use a word,' Humpty Dumpty said, in a rather scornful tone, 'it means just what I choose it to mean, neither more nor less.'*

From *Through the Looking Glass* by Lewis Carroll

The new *Primary framework for literacy and numeracy* has recently been launched in electronic form on the Standards website (1). The mathematics section comprises the following sub-sections: guidance papers on calculation, calculators and using and applying mathematics; learning objectives; assessment; planning; and problem solving, reasoning and numeracy from the Foundation stage. This article focuses on one particular aspect of some of the papers relating to Year 1 children.

One issue that the original National Numeracy Strategy was quite pernickety about was the use of correct mathematical language - even to the extent of producing a vocabulary book setting out which words should be known and used in each year. Teachers were expected to 'highlight the meaning of any new vocabulary, notation or terms, and get pupils to repeat these and use them in their discussions and written work' (2). Additionally, many of the examples of good practice featured in official NNS videos show teachers referring to posters that list the key vocabulary for the lesson.

So, imagine my surprise when reading the revised framework to encounter (amongst the many improvements) what seemed to be a very idiosyncratic use of the phrase 'count on'. As mathematics educators know, 'counting on' is an apparently simple (but actually, quite sophisticated) strategy used for finding the answer to an addition problem. Young children often progress to it after using the self-explanatory 'counting all' strategy for a brief period. It comes in two forms: 'count on from first' and 'count on from larger'. An example of the latter strategy to find, say,  $5 + 8$  would be: "Eight...nine, ten, eleven, twelve, thirteen. It's thirteen." Notice that in counting on, you recite the number names that are the successors of one of the numbers, whilst simultaneously keeping track of how many number names you have spoken. When counting on, you know how many number names you have to recite (in this case 5), but you do not know what the final number name (i.e. the 'answer') is going to be.

What has always amazed me about this so-called low-level strategy is that the child actually makes a cardinal/ordinal switch followed later by an ordinal/cardinal switch: in the example above, the cardinal number 5 in  $5 + 8$  is transformed into an ordinal number so that the count can be continued to 13; then the ordinal number 13 is converted back to a cardinal quantity to provide the answer. And yet some people call it a basic strategy!

One of the early statements in the outline for Year 1 is:

They count on in tens starting from zero or from a multiple of 10 such as 40, and count back to zero.

This situation does not match the counting on situations described above. Here the children are just being asked to 'continue the count in tens...', or more simply, 'count in tens...'.

In a section about relating addition to counting on and combining groups, we are informed that children initially use practical apparatus to model a problem and:

Later, they count on using a number line, then count on mentally. I can find no evidence in any document of an attempt to make teachers and trainees aware of the conceptual difference between counting on using a number line and counting on mentally. For example, try the following: imagine a number line, and take note of the words you would say (perhaps sub-vocally) when using counting on to calculate  $5 + 4$  on this line. Then use counting on to find  $5 + 4$  in your head - again taking note of the actual words that you use. Finally, compare the two lists of number names and other aspects of the two processes.

I assume that most of you moved along the line saying; 'One, two, three, four', noting that you landed on position 9. (Even if you did not, my experience suggests that the majority of people do). In the second situation you most probably said: 'Five... six, seven, eight, nine', noting that the last number name spoken was 'nine' - the answer to the addition. In these two situations you recited different number names when carrying out the activities, and ascertained the answer from different contexts - the position on a line and the last word spoken. This suggests that they are actually different strategies, even though, and unfortunately to my mind, we use the same words to describe them. ('Counting along' is more accurate in a number line context - although it is much more clumsy).

Elsewhere in the documents we find the following statement:

They use a bead string or a number line to work out calculations such as  $8 + 5$  or  $18 + 5$  by counting on, using 10 and 20 as milestones, for example  $8 + 2 = 10$  and  $10 + 3 = 13$ .

The procedure described here does not involve counting on, since the child is adding 2 to reach the 10 milestone and then adding 3 more to reach the total. If the child were indeed counting on, i.e. saying 'Nine, ten, eleven...' then 10 would be no more of a milestone than 11 or 12. A more accurate description of the strategy is 'adding on' rather than 'counting on', although the process should be correctly described as 'bridging' - in this case 'through ten'.

Researchers have used the term 'bridging' for many years to describe this very important strategy, which comes into its own with two-digit mental addition and subtraction. The original framework also used the term bridging, but described it as 'bridge through 10 and adjust'. With bridging there is actually no adjustment to be made, just a continuation of the calculation. This is different from 'compensation', where you do have to make an adjustment according to the extra amount that you have added or subtracted. The revised framework appears to have dropped the 'and adjust' phrase when describing bridging, but has retained it for the addition or subtraction of near-multiples of ten. It is a pity that 'compensate' was not used in this context rather than 'adjust', as this would have maintained a link with the nomenclature used for mental calculation in the rest of Europe.

When bridging, you add part of one addend to make the other into a multiple of ten in order to simplify the next part of the calculation, and then you add the rest to get the final answer. It is interesting to observe that English text books, even those written post-NNS, tend to focus on asking children to work with number sentences of the form  $6 + 3 =$ . European textbooks, on the other hand, espouse the principle that it is more important to be able to find some or all of the solutions of  $9 =$ , given that one important pre-requisite of bridging is the ability to partition single-digit numbers accurately.

The Year 1 outline also informs us that:

Children build on their understanding of subtraction to interpret  $14 - 9$  as... 'How many more must I add to 9 to get 14?' They use a counting on strategy and record the process as steps on a number line.

This is another misuse of the phrase 'counting on'. To recapitulate, 'counting on' is a strategy that is used for solving addition problems: you count without knowing where you are going to finish your count. However, what you do know is that when you have recited the appropriate number of counting words, the last number name you say will be the answer to the calculation. In the 'How many more must I add...' situation described above (which is actually 'complementary addition', but not named as such in this overview) you *do* know where you are going to end up, i.e. at 14. What you do *not* know is how many steps you will have to take to get there. This is, obviously, a different situation, and for that reason we give the strategy a different name: viz.

'counting up'. The original framework used this phrase correctly on every occasion, so how have these incorrect modifications been introduced?

This information has been available in research papers for many years, and has been made much more accessible since 1997 (3). Also, given that some researchers in the area of mental calculation (4, 5) advocate children using the names of calculation strategies in their class discussions as they do in some Dutch schools, the sloppy use of technical terms in these revised framework documents does not augur well for such a recommendation ever being adopted here.

Unfortunately, there is no space here to discuss 'counting back', given the two different mental procedures in addition to the number line strategy!

#### References

- 1 <http://www.standards.dfes.gov.uk/primaryframeworks/>
- 2 DfEE: *The National Numeracy Strategy: framework for teaching mathematics from Reception to Year 6*, London: DfEE, 1999.
- 3 I Thompson: *Teaching and learning early number*, Open University Press, Maidenhead, 1997.
- 4 S. Higgins: Parlez-vous mathematics? in I. Thompson (ed), *Enhancing primary mathematics teaching*, Open University Press, Maidenhead, 2003.
- 4 M. Beishuizen: The empty number line as a new model, in I. Thompson (ed), *Issues in teaching numeracy in primary schools*, Open University Press, Maidenhead, 1999.

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