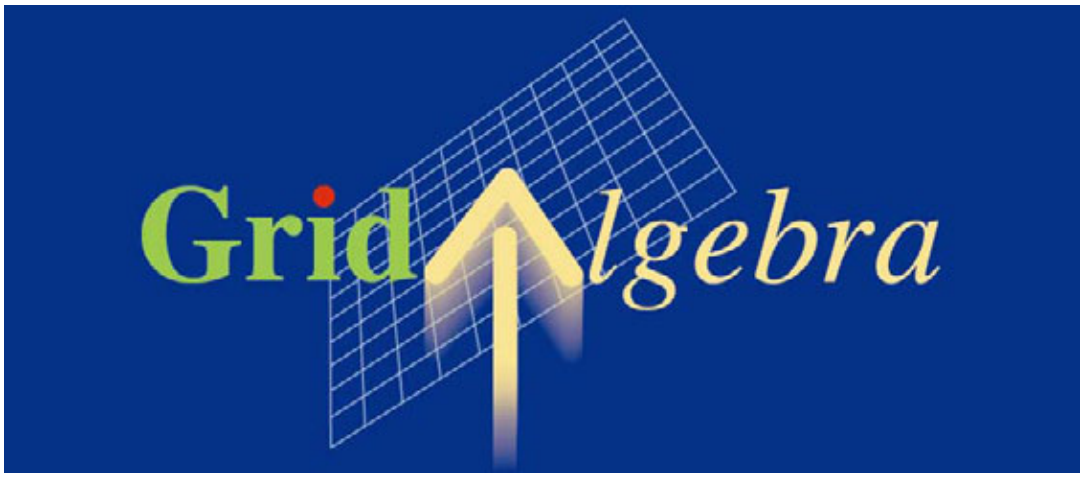
Grid Algebra

A description of the software



Developed by Dave Hewitt
Programmed by Paul Hayton

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Note: if printing this document it is preferable to set the print settings to 'best quality' otherwise some lines in the grids of the screen shots within this document may not be printed.

Installing the software

Installation on a single PC

This CD-Rom will install automatically on most PCs. To use this CD-Rom for the first time:

1. Insert the CD-Rom into the disk drive.
   If Auto Install is disabled, run the installer program Setup.exe in the root directory of the CD.
2. Follow the on-screen instructions provided by the setup program.

For RM Network Installation

Follow the steps in the RM network Reference Manual Chapter 7 to create a package. You can find the Windows installer file for Grid Algebra by selecting the CD-ROM, right clicking and selecting Explore.

Basic system requirements

Hardware	An Intel® Pentium® class or AMD-K6® class processor with at least 128MB of RAM
Software	Microsoft® Windows® 2000/XP or Windows NT® 4.0 with Service Pack 4 or later.
Disk Space	Approx 100 MB for full installation.

Acknowledgements

Creating software is a team effort:
Dave Hewitt has been the originator and leader of this project. We would also like to thank Paul Hayton (programmer), Jenni Ingram and the members in the ATM ICT group, the ATM Publications group, and Karen Moran in the ATM office, Olivia and the many other children and teachers who have tried the software and provided feedback.

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About Grid Algebra

Grid Algebra is based on movements around a grid of multiplication tables. It relates the everyday awareness children have about movement to their mathematical awareness in number and algebra.

- *Same place* spatially relates to *equivalent* mathematical expressions.
- Each *movement* made relates to a corresponding *operation* within a mathematical expression.
- *Order of movements* spatially relates to *order of operations* mathematically.
- *Inverse movements* spatially relate to *inverse operations* mathematically.

Grid Algebra offers images to help with number work in primary and secondary schools, including *multiplication tables, mental arithmetic, multiples, factors, negative numbers* and *equivalence*. It helps introduce algebra work, where the appearance of a letter and the building of expressions involving a letter can happen relatively seamlessly. Expressions are not seen as an abstract collection of symbols but have a visual and kinaesthetic meaning for pupils. Algebra work such as *reading the order of operations, finding inverse operations* and *solving equations* can become more intuitive. Other algebra topics such as *simplifying*, *substituting*, *expanding* and *factorising* are linked to the imagery of the grid.

Grid Algebra includes lesson ideas to use with the program, computer generated tasks for students to engage with and a collection of handouts for working away from the computer.

Interactive Grid Algebra includes all the facilities for you and your pupils to explore work on the grid and devise your own ways of using the software.

Using Grid Algebra

The Main Menu screen shows the three parts of the program: Interactive Grid Algebra, Tasks and Resources. These are briefly described below with more detailed information later on.

The Main Menu screen of Grid Algebra

Interactive Grid Algebra is the main software with full functionality for you to explore the grid, place numbers and letters and create your own expressions and equations. You can design grids in advance to engage with particular activities or use one of the many lesson ideas in the resources section.

Tasks are a collection of 26 computer tasks which cover a variety of topics in number and algebra and address a range of difficulty.

Resources has a collection of lesson ideas for using Interactive Grid Algebra, each of which has prepared grids and handouts for students. This section also contains information about using the software (including this document!).

Help

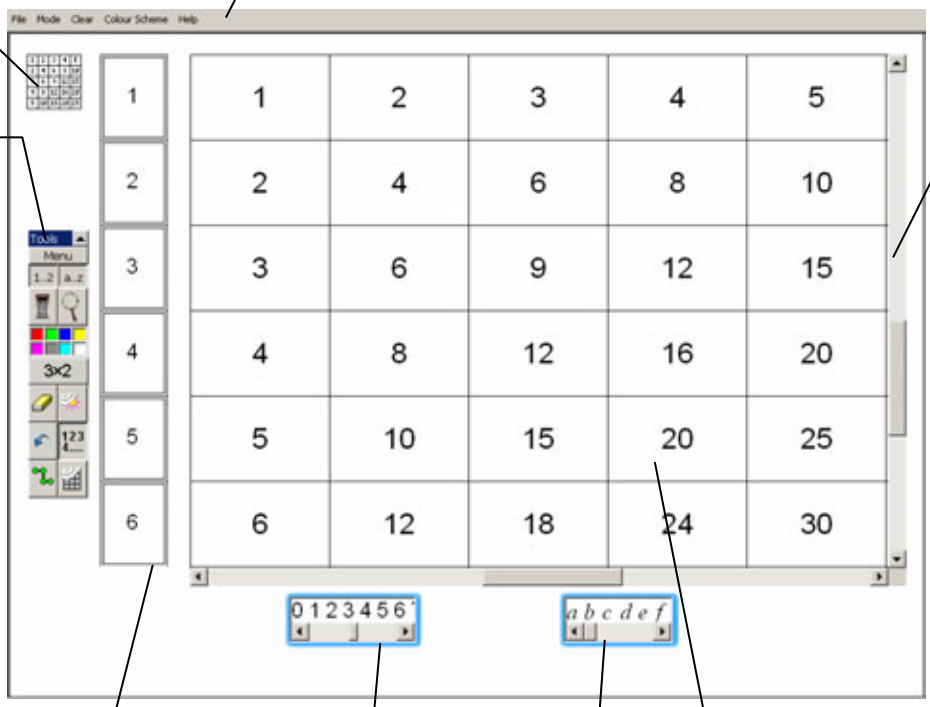
Help with using the software can be found through one of three sources:

- the booklet accompanying the software – this gives a good general overview;
- a more detailed description of the software (you are reading it now!) which is found in the General Resources section of the Resources screen;
- the 'demos' which can be found under the Help menus in Interactive Grid Algebra and each of the Tasks.

Interactive Grid Algebra – The screen

Grid defined indicator: when numbers inside all cells of the main grid are defined this icon will show numbers inside it (as it does now). When the main grid is not defined this indicator is blank.

Menus: for File (new, load, save, exit); Mode (full screen on/off); and Clear (all, numeric only, algebraic only); Colour Scheme (black on white, white on blue, black on grey); and Help (about and demos). Note that demos are short films which describe an overview of Interactive Grid Algebra (when you are in this screen) or each of the separate Grid Algebra Tasks (when you are in a Task screen). If you are in need of help go to the Demo which can be found under the Help menu.



Toolbar (see separate page for more information).

Scroll bar: to move to different areas of the grid.

Row indicators: tells you the times table which appears in each row.

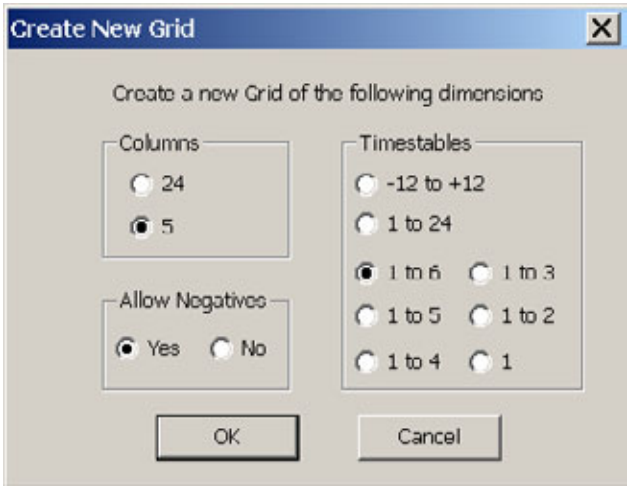
Number Box: numbers can be dragged into the grid. Note this box only appears when the 'Numbers' button in the toolbox is pressed down.

Letter Box: letters can be dragged into the grid. Note this box only appears when the 'Letters' button in the toolbox is pressed down.

Grid: based upon rows of multiplication tables. At present there are numbers in all the cells but there might be no numbers at all or only the numbers or letters you have placed in the grid. Numbers placed in the grid are only accepted if they fit in with the overall multiplication table structure.

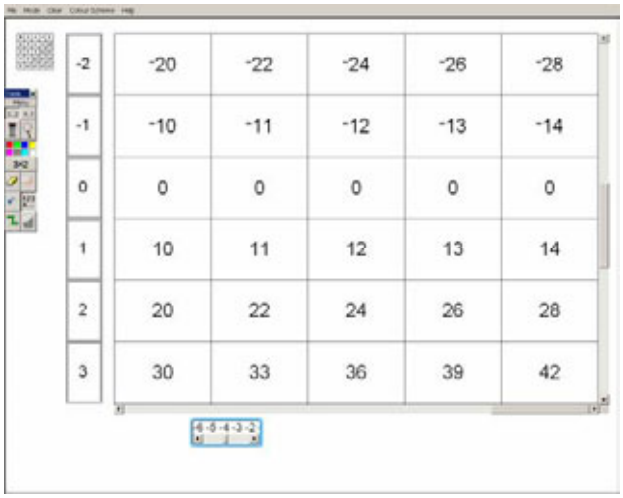
Interactive Grid Algebra – The grid

The grid can be changed to suit the activity. The grid is always based on the multiplication tables, but the number of tables and the number of columns can be chosen, as can whether negatives numbers are available. Through the File menu and choosing New Grid, a box appears as below.



Examples of grids are below. The grid on the left has just the 1 to 2 times table and only 5 columns. The grid on the right has the -12 to 12 times tables and 24 columns. At any one time you can see a 5 by 6 part of the overall grid. You can scroll to look at different parts of the full grid. Some scrolling has taken place in this screen shot.

A grid with just the 1 and 2 times tables

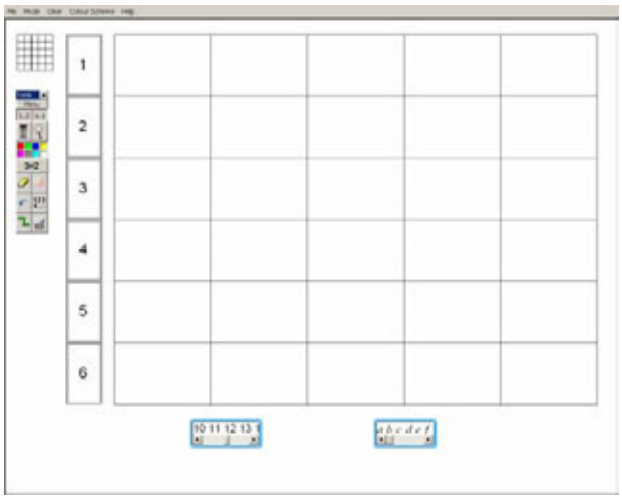


A grid with -12 to 12 times tables

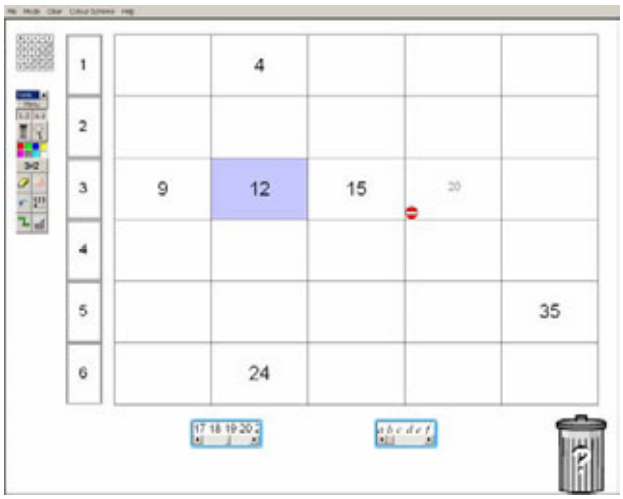
Allowing negatives in the New Grid box above will mean the Number Box, which can be seen at the bottom of each screen shot, will have negative numbers in it. This is the case in the right hand screen. The left hand screen has not allowed negatives and so the lowest number in the Number Box is zero. See the next page to find out more about using the Number Box.

Interactive Grid Algebra – Placing numbers and letters in the grid

These screens are about placing numbers in the grid.



An empty grid



Numbers placed in the grid

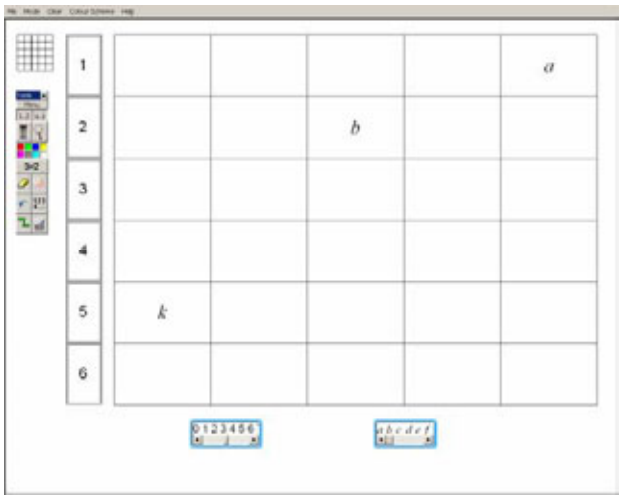
You can drag a number from the Number Box at the bottom of the screen and drop that number into a cell in the grid. However, a number will only be accepted if it fits in with the times table structure of the grid. Each row is a times table. In an empty grid (see above left) the number 12 can be dragged into any cell in rows 1, 2, 3, 4 and 6 because 12 is in those times tables. However, it will not be accepted in row 5 as it is not in the 5 times table.

Once 12 is placed somewhere (as in the highlighted cell in screen above right), then the rest of the grid is now defined. Notice the grid defined indicator at the top left of the screen has numbers inside to indicate this. Now the grid is defined there will only be one number accepted into any of the other cells. The numbers 9 and 15 must go either side of the 12 because it is in row 3, the 3 times table. I have tried to put the number 20 next to 15 in row 3 but this has been rejected. When you see the bin you can click on the bin and it will tell you why something has been rejected.

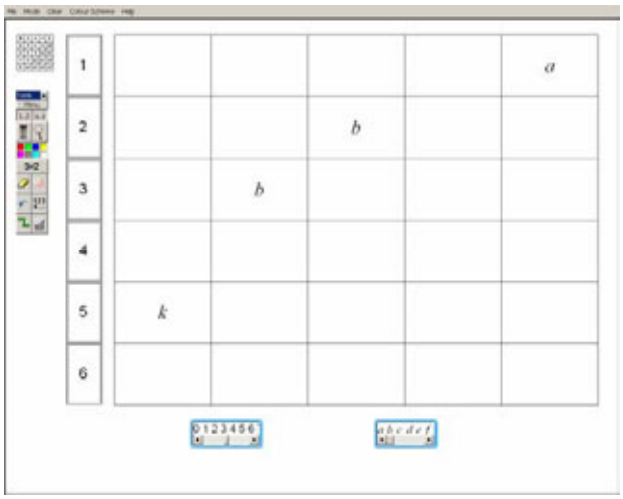
Since 12 is 4x3 then the number 4 must go in row 1 above the 12. Also number 24 must be in the same column in row 6 as 12 is in row 3. Convince yourself why the number 35 must be in the cell it is in!

For programming reasons there is a restriction on the size of number which can be placed in any cell in an empty grid. You will not be allowed to put any number above 50 in row 1 when the grid is empty. For other rows, say row *n*, the restriction is 50*n*. So for row 3 the maximum number allowed to be entered in row 3 will be 50x3=150 when the grid is empty. Likewise it will be 50x24=1200 for row 24 (the grid can go up to 24 rows). However, the Number Box only has numbers up to 200. For entering higher numbers use the Expression Calculator (see page 13).

These screens are about placing letters in the grid.



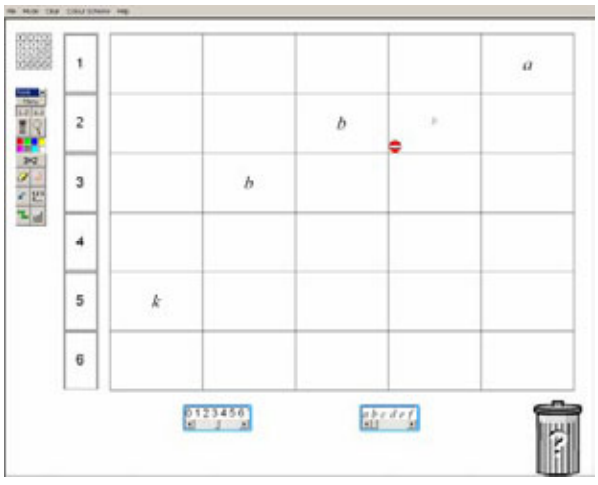
Letters placed in grid



Two copies of the same letter in the grid

When the grid is empty a letter can be dragged from the Letter Box and put into any cell. It does not matter which cell as the letter starts off not having any particular value (a variable). Once it has been placed somewhere, such as the letter *b* in the top left grid, then it now must be a number in the 2 times table as it is in row 2. Notice the grid defined indicator at the top left of the screen is blank showing that the grid is not defined as *b* could be any multiple of 2.

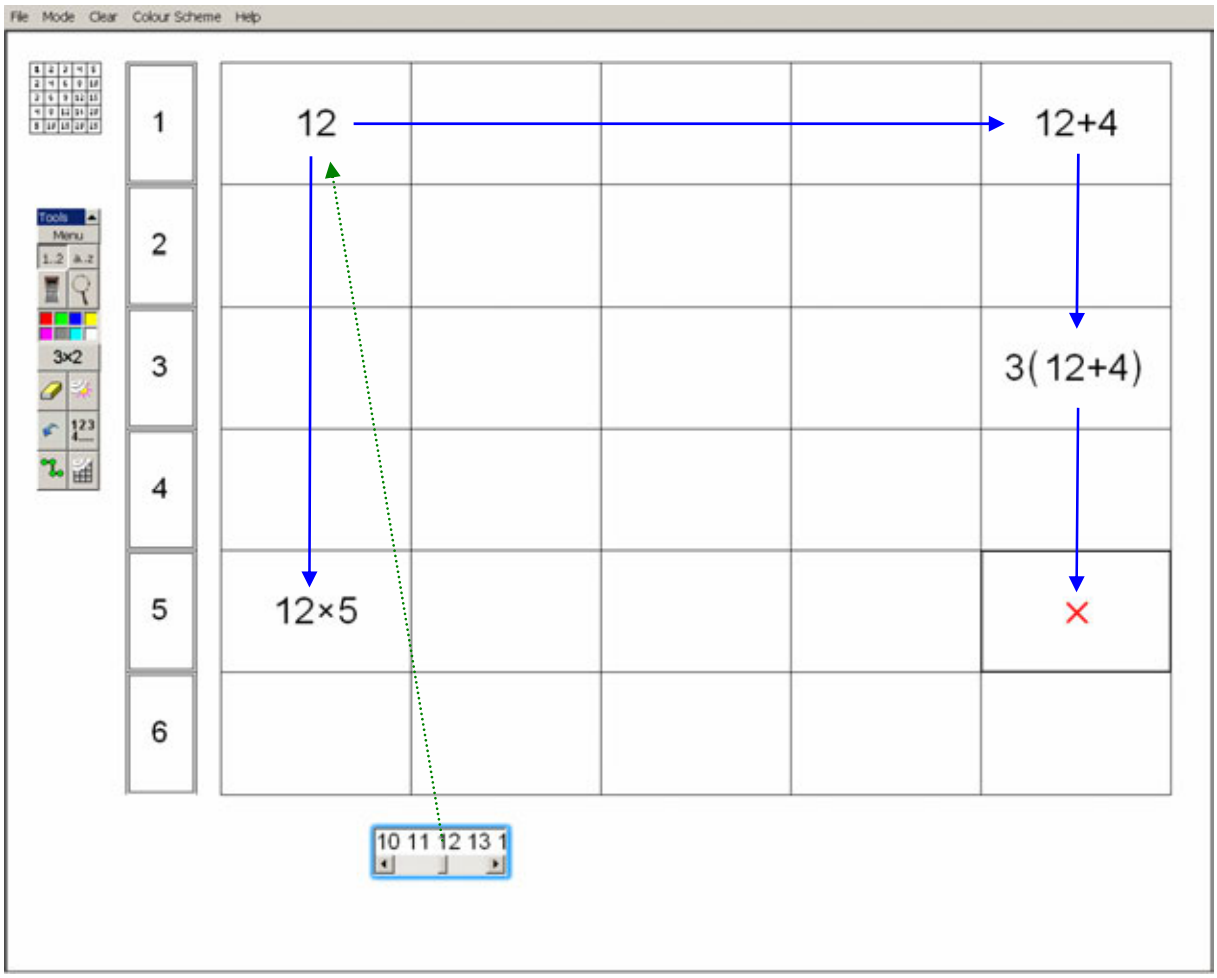
In the top right grid a second *b* has been placed in a different cell (note that the second *b* could not have been put into any cell – This is a focus for one of the Tasks!). With the same letter now in two cells this means those two cells must be the same number. The grid defined indicator is now full of numbers showing the grid is now defined. So *b* has shifted from being a variable to being an unknown. It must be a number in the 2 and 3 times tables and I will leave you to work out why it must be 6. The other letters now have definite values as well. I will leave you to work these out! In the grid below I have tried to place a third *b* but it has been rejected. One reason is that there cannot be two numbers the same in the 2 times table.



An example of when a letter is rejected

Interactive Grid Algebra - Movements around the grid

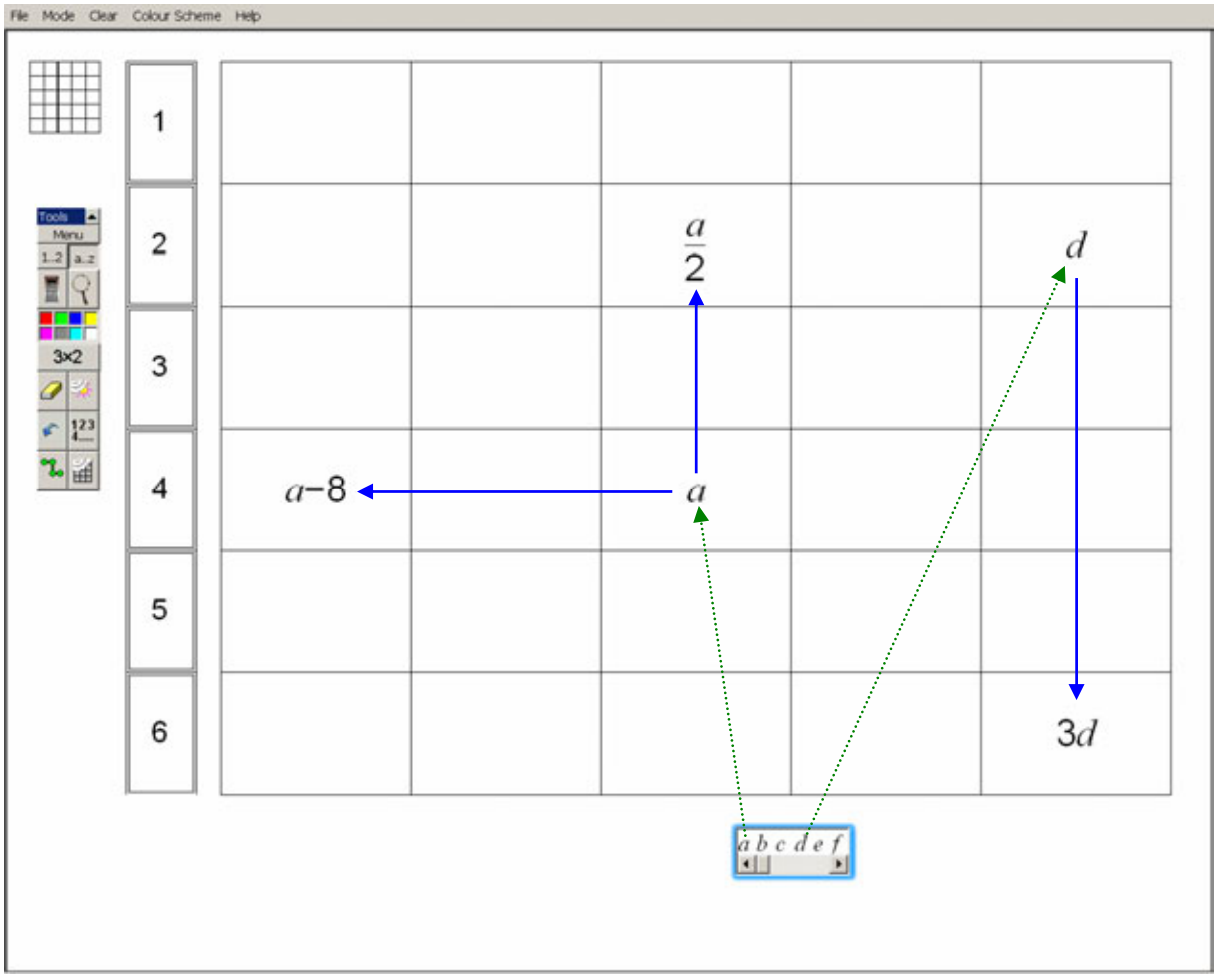
This screen shows the movement of a number around the grid.



Movement of numbers around the grid

- Number 12 has been dragged from the *Number Box* and dropped in the top-left cell.
- Then 12 has been dragged from top left cell in Row 1 (1 times table) to Row 5 (5 times table). This multiplies 12 by 5.
- Also 12 has been dragged four cells to the right in row 1. This adds 4 to 12.
- The result 12 + 4 has been dragged down to row 3. This multiplies 12 + 4 by 3.
- 3(12+4) in row 3 cannot be dragged down to row 5. To keep things simple Grid Algebra only allows whole numbers to be used. Moving from the 3 to the 5 times table would involve multiplying by a non-integer and is not allowed.

This screen shows the movement of letters around the grid. Letters can be moved in exactly the same way as numbers. This is one reason why it can be relatively straightforward to introduce and use letters with Grid Algebra.

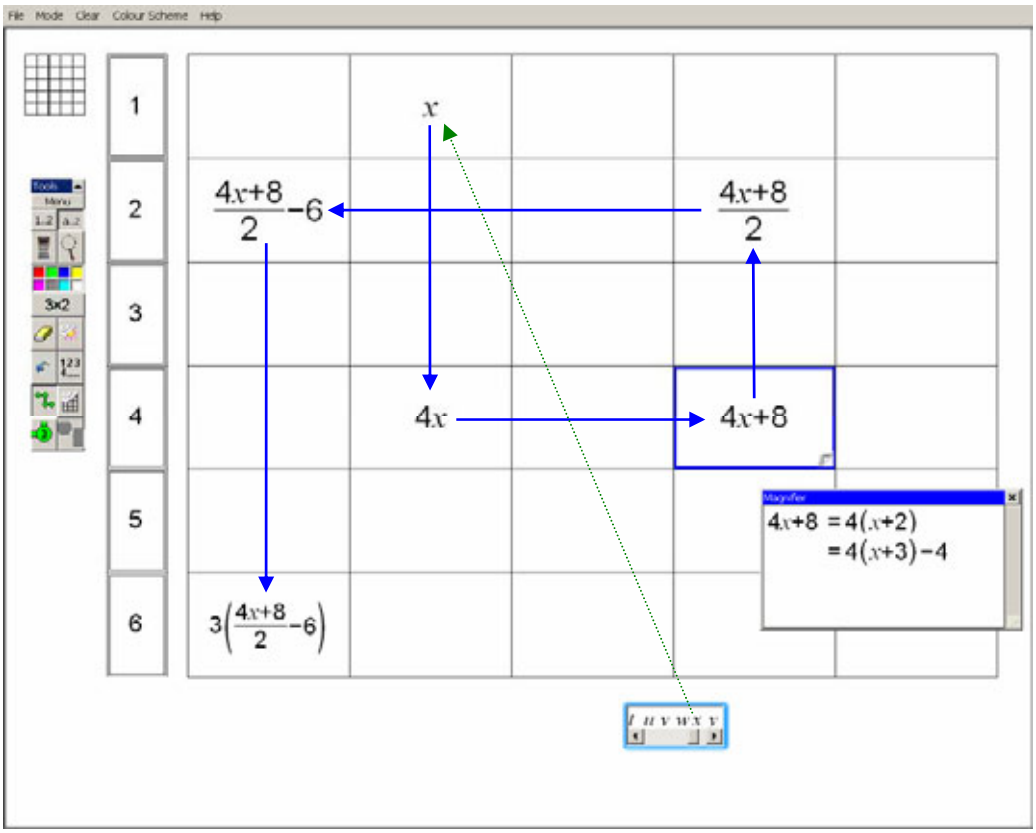


Movement of letters around the grid

- Letters *a* and *d* have been dragged from the *Letter Box* and dropped into cells.
- Letter *a* has been dragged from Row 4 (4 times table) to Row 2 (2 times table). This divides *a* by 2;
- *a* has also been dragged 2 cells to the left in Row 4 (4 times table). This subtracts 8 from *a.*
- *d* has been dragged from Row 2 (2 times table) to Row 6 (6 times table). This multiplies *d* by 3.

Interactive Grid Algebra - Building up an expression

Expressions involving a number of operations can be built up through dragging a number or letter to another cell to make an expression such as $4x$ (see grid below). Then picking up that expression and carrying out a new movement on that. This can be continued to build up quite complex expressions should you wish to do so! The font size changes automatically to fit the expression into a cell but a magnifier (see later) can be used to show everything in a particular cell in a large font.



An expression being built from a series of movements

- Letter x has been dragged from the *Letter Box* into a cell.
- The arrows indicate the journey taken. There are five movements on the grid which relate to five operations in the final expression.
- In row 2 (2 times table), for each cell moved left, 2 is subtracted. In row 4 (4 times table) for each cell moved to the right, 4 is added.
- The Magnifier in the highlighted cell shows all expressions in that cell. In the case shown, there have been two other journeys made from x to that cell.
- All windows, such as the magnifier, can be moved by dragging the top bar. They can also be re-sized by dragging the bottom right corner. This also affects the font size in the window.

Grid Algebra Interactive: the toolbar

Numbers: opens and closes the Number Box.

Minimises and maximises the toolbar.

Menu: click to get menu choices: new grid, save and load file, clear the grid and exit.

Expression Calculator: click and mouse will change to an icon. Then click in a cell and the Expression Calculator will appear. Use it to enter an expression into that cell.

Letters: opens and closes the Letter Box.

Highlight: enables you to change the background colour of a cell.

Magnifier: click and mouse changes to icon. Then click in a cell to see all expressions in that cell.

Rubber: enables you to rub out expressions from the grid.

Notation: change the way multiplication of numbers is shown.

Hide/show: when pressed down click on expressions to put them in hidden or shown mode. When pressed back up again all expressions in hidden mode cannot be seen.

Inverse: this will have no effect unless you drag an existing expression in such a way that the last operation in that expression is undone, in which case that last operation will disappear from the expression.

Fill Grid: fills grid with numbers when pressed down. When pressed back up the filled numbers disappear.

Route: press down to see existing route or to create a new route (two additional route buttons will appear as below). When this button is pressed up again the existing route will disappear.

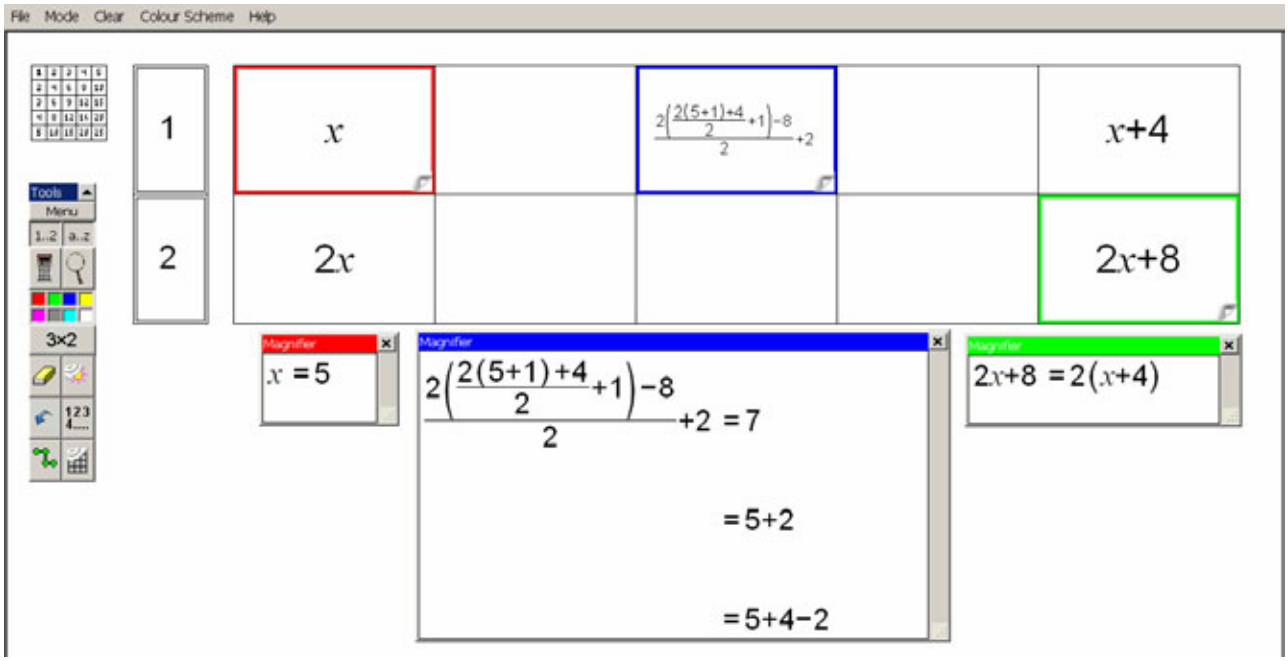
Hide Grid: when pressed down grid is hidden and only a calculator, magnifiers, route window or equation window can be seen. Press up to show grid again.

New Route: press down to begin creating a new route. Then click on cells to define the route. When finished press this button up again.

Expression Window: when a route is created on the grid press this to show the expressions along the route. There will be an option to change this into an Equation Window if the final cell in the route is a single letter/number. This will then show the original and inverse route together as a series of equations.

Grid Algebra – Peeled back corners and Magnifiers

Here is a screen with just 2 rows where letters and numbers have been dragged around and three magnifiers have been put into cells.



A 2 row grid with Magnifiers placed in three cells

Peeled back corners

In the top left cell you can see the letter x. At the bottom right of that cell is a peeled back corner. This indicates there is something else in that cell as well. If you click on this corner you scroll through all the expressions in that cell. The Magnifier below the cell shows everything in that cell. In this case there is x and the number 5.

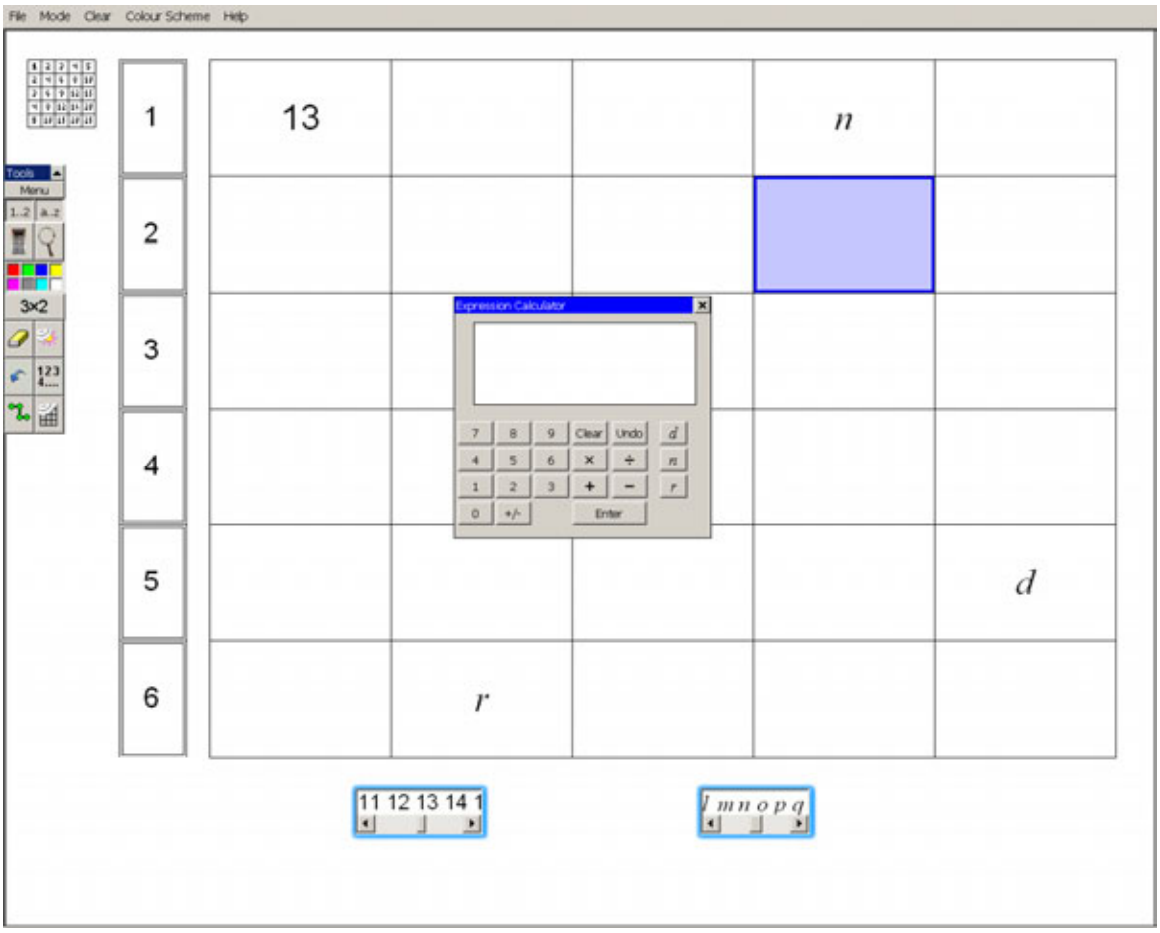
Magnifiers

A magnifier has been placed in the top left cell to reveal both x and 5 in that cell. All expressions in the same cell are equivalent. So this is shown in the magnifier as $x = 5$. Two other magnifiers have also been put into cells. Each magnifier is coloured the same as the cell to which it relates. Magnifiers can be dragged to a convenient position by clicking on the top bar of the magnifier and dragging.

The middle magnifier shows there are four expressions in that cell. This magnifier has been re-sized to be bigger by clicking on the bottom right corner and dragging. Re-sizing a magnifier can change the size of the font inside. So this is a way of making some expressions bigger.

The order of the expressions within a magnifier can be changed by clicking on one of the expressions. That expression will then move to the top in the cell and also appear on the left-hand side of the equals sign in the magnifier.

Grid Algebra – The Expression Calculator



The Expression Calculator placed in a cell with buttons for the letters *d*, *n* and *r*

If there are letters in the grid when the Expression Calculator is dropped into a cell, then those letters become available as buttons in the Expression Calculator. In the case above buttons have appeared for *d*, *n* and *r* in the Expression Calculator.

Order of pressing buttons

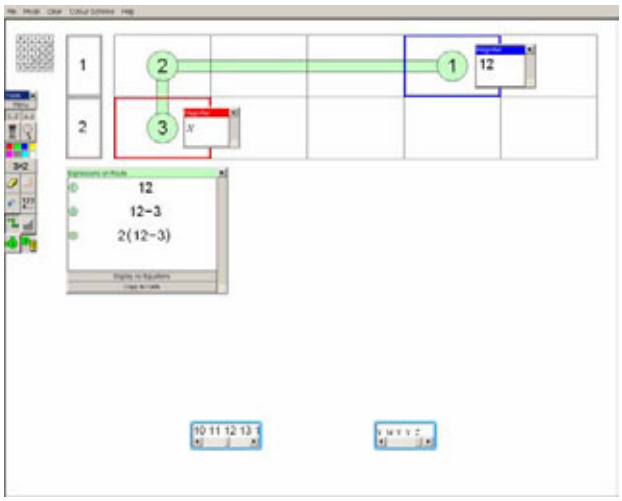
One journey from 13 to the highlighted cell is going three cells to the right and then down. This would produce the expression 2(13+3). All expressions entered into the Expression Calculator must be entered in order of operations and not left-to-right. So the order of buttons pressed would be: 13 + 3 x 2. This is deliberate to help students get used to seeing and reading the order of operations within a formal expression. A journey from *n* down to the highlighted cell is 2*n*. The order of buttons pressed would be *n* x 2. If you press 2 first you would find the letter buttons would no longer be available as in the grid any expression including a letter would have started with that letter.

Entering negative numbers

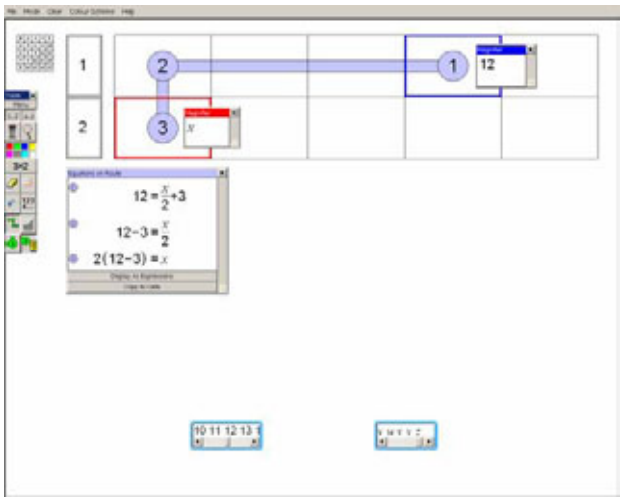
Negative numbers are entered using the +/- button. For example, -7 would be entered by pressing buttons in the order 7 followed by +/-.

Grid Algebra – Routes

Routes can be drawn on the grid as in the screens below.



A route with Expression Window



The same route with Equation Window

Drawing routes

Looking at the screen on the left above, the route was drawn by clicking on the Route button on the toolbar. This makes another two buttons appear at the bottom of the toolbar. The New Route button is then pressed and you click on the cells in order to create your route (you will not be allowed to start a route from an empty cell). The green route will appear as you do this. When you have finished, press the New Route button up again to indicate you have finished. The route markers sometimes hide all or part of an expression in a cell and so I have put Magnifiers into the start and end cells to show that 12 is in the start cell and x is in the end cell.

Expression Window

Click on the Expression Window button in the toolbar to get up a window showing all the expressions which would appear along the route if the number 12, in this case, was dragged along the route. There is a button at the bottom of the Expression Window to copy these expressions into their cells if you wanted to do that.

Equation Window

If the cell at the end of the route has a single letter or number showing as you create the route (which in this case it does as the letter x was there), there will be a button in the Expression Window saying 'Display As Equations'. If this is pressed the screen will change to that on the right above. Now it will show not only the original expressions along the route starting at 12 but it also shows the expressions from the inverse route going backwards starting at x. This forms a series of equations similar to those involved in solving an equation.

Resources

The Resources screen with the topic Order of Operations chosen

The resources section has lesson ideas, prepared grids, handouts and information about using Grid Algebra (including what you are reading now!).

Lesson ideas
These are organised under topic headings and difficulty level. Click on a topic, such as Order of Operations in the screen above, and all the lesson ideas relating to that topic will be available from the drop down menu. These lesson ideas explain how you might use Interactive Grid Algebra to approach that topic with a class. They all have a prepared grid which can be used at the start of the lesson. Alternatively click on a difficulty level and all the lesson ideas relevant to that difficulty level will be available from the drop down menu. More details about lesson ideas are on the next page.

Prepared grids
Interactive Grid Algebra can have its grid specially prepared and saved to be ready for a lesson. All lesson ideas have at least one prepared grid which can be used for that topic. The prepared grids can be started by selecting the appropriate one from the drop down menu. Interactive Grid Algebra will then automatically open up with the prepared grid ready for use.

Handouts
These are files ready to be printed off and handed out to a class. Each lesson idea has at least one handout for follow up work.

General Resources
These include information about using Grid Algebra, along with prepared grids and handouts which can be used for a variety of topics and difficulty levels.

Lesson ideas linked to topics and difficulty levels

This chart links the lesson ideas with topics and difficulty levels.

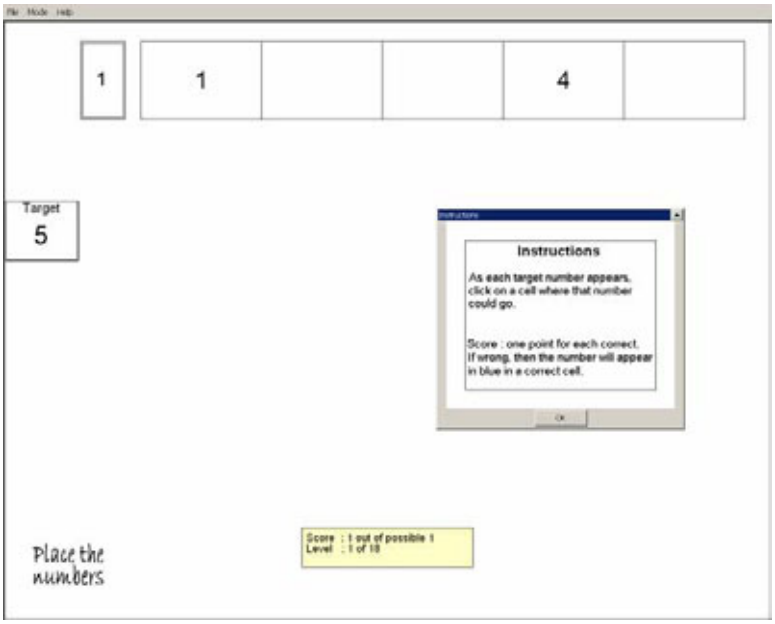
Topic \ Lesson idea	Different ways to make a number	Equivalent expressions	Expanding and factorising	Factors	Getting to know the grid	Inverse journeys	Journeys	Multiples	Negative numbers adding and subtracting	Negative numbers multiplying and dividing	Simplifying	Solving equations	Solving step by step	Substitution	Where are the letters?	Which number goes here?
Multiplication tables								X								X
Multiples								X								
Factors				X												
Negative numbers									X	X						
Formal notation		X			X	X	X						X		X	
Equivalent expressions	X	X														
Order of operations						X	X					X	X		X	
Substitution														X		
Simplification											X					
Inverse						X						X	X		X	
Solving equations												X	X			
Expanding		X	X													
Factorising		X	X													
Difficulty level																
★	X			X	X			X			X					X
★★	X	X		X	X	X	X	X	X		X			X	X	X
★★★	X	X	X	X	X	X	X		X	X	X	X	X	X	X	X
★★★★	X	X	X		X	X	X			X	X	X	X	X	X	X
★★★★★						X	X				X	X	X		X	

Tasks

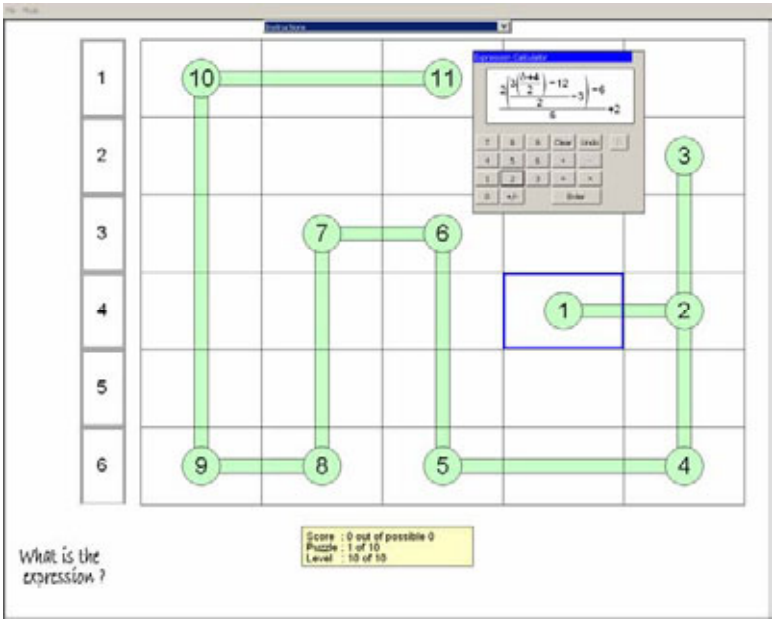
General description

There are 26 structured, computer-generated tasks to engage pupils, individually or in pairs using a computer or with the whole class using an interactive whiteboard. The tasks cover a range of number and algebra topics. Within a single task there are many levels of difficulty. Each level will have either 10 or 20 puzzles, so there is a lot of possible student activity!

The range of tasks and levels will suit primary pupils up to older secondary pupils. An example of the range of difficulty is shown below.



Screen shot of a task at the lowest level of difficulty



Screen shot of a different task at the highest level of difficulty

Tasks and their difficulty ranges

There are 26 tasks and they are ordered in alphabetical order. The stars indicate the difficulty level range from the first to the last level of puzzles in each task:

	Task	**Difficulty range**
1	*Calculating* Find the value of a given number expression.	★ to ★★★★
2	*Calculating – negative numbers* Find the value of a given expression involving negative numbers.	★★ to ★★★★
3	*Calculating – small grids* Find the value of a given number expression.	★ to ★★★
4	*Equivalent expressions (letters)* Enter 3 equivalent journeys for each situation.	★★ to ★★★
5	*Equivalent expressions (numbers)* Enter 3equivalent journeys for each situation.	★ to ★★★
6	*Expanding Factorising* Expand or factorise given expressions.	★★★★to★★★★★★
7	*Find the journey (letters)* - on large grid Find the journey for a given expression.	★★ to ★★★★★
8	*Find the journey (letters - small grids)* Find the journey for a given expression.	★ to ★★★★
9	*Find the journey (numbers)* - on large grid Find the journey for a given expression.	★★ to ★★★★★
10	*Find the journey (numbers - small grids)* Find the journey for a given expression.	★ to ★★★★
11	*How many places?* Find all the places on the grid where a given number should appear.	★ to ★★
12	*Inverse journey* Enter inverse journey given original journey.	★★★★to★★★★★★
13	*Make the expression (letters)** - on large grid Move round grid to create given expression.	★★ to ★★★★
14	*Make the expression (letters - small grids)** Move round grid to create given expression.	★ to ★★★
15	*Make the expression (numbers)** - on large grid Move round grid to create given expression.	★★ to ★★★★
16	*Make the expression (numbers - small grids)** Move round grid to create given expression.	★ to ★★★

17	*Make them equal!* Find the number which will make two given cells have the same value.	★★★★to★★★★★
18	*Multiple inverse journeys* Match up an expression with its inverse.	★★★ to ★★★★
19	*Place the numbers (single player)* Find a cell where a given number could go.	★ to ★★★★
20	*Place the numbers (two players)* 2-player game placing numbers on the grid	★ to ★★★★
21	*Simplify* Enter simpler expression equivalent to expression given.	★★ to ★★★★★
22	*Substitution* Value when substituting a number for a letter.	★★ to ★★★★★
23	*Substitution – negative numbers* Value when substituting a number for a letter	★★★ to ★★★★★
24	*Substitution – small grids* Value when substituting a number for a letter.	★★ to ★★★★
25	*What is the expression?* Enter the expression for a given journey.	★★ to ★★★★★
26	*Where is the letter?* Given an expression, find cell containing letter.	★★ to ★★★★★
	** indicates a timed challenge*	

There is a separate document giving more details about these tasks. This can be found in the General Resources section on the Resources screen.

Grid Algebra – Some ideas behind the program

The Grid

Grid Algebra is based on a grid of multiplication tables.

1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25
6	6	12	18	24	30

The grid is based on the multiplication tables

There are many ways in which this grid is used for number work within the program. However, the shift from number into algebra comes from stressing operations rather than numbers themselves. This is achieved through considering journeys between the numbers on the grid. For example, moving from the number 3 to the number 5 in row 1 involves adding 2; moving from the number 3 in row 1 down to the number 12 in row 4 involves multiplying by 4. In Grid Algebra you can dynamically drag a number from one cell to another and the operation will be automatically shown as you move. So in the examples I have given 3+2 will appear in the same cell as number 5 in row 1 and 3x4 will appear in the same cell as number 12 in row 4. Through a succession of movements an expression involving several operations can be created.

The expressions can get quite complex but since they have been created through physical and visual movements it helps students develop meaning for those expressions by seeing the expression as a historical record of the movements carried out.

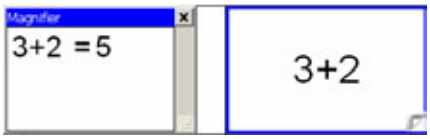
Equivalence

Grid Algebra uses a natural sense of sameness where those things which end up in the same place are equivalent. 3+2 ends up in the same place as 5 and these are equivalent mathematically.

Same place spatially relates to *equivalent* mathematically.

When there is more than one expression in a cell a peeled back corner indicates that there is more in that cell.

A magnifier can be put into the cell to show everything in that cell.



Notation

Grid Algebra shows in mathematical notation the movements someone has done on the grid. The movement from the number 3 to the number 5 in row 1 is shown as 3+2. If this new expression is then picked up and moved vertically down to the number 20 in row 4 then 4(3+2) would appear in that cell. The notation represents the two movements.

Each *movement* made relates to a corresponding *operation* within a mathematical expression.

The notation carries within it the historical story of the movements which have been carried out. The notation is both an *object* which represents a cell and which can be picked up and moved, and a *process* as it represents the historical record of what has been carried out. This dual representation of notation as both process and object helps with later algebraic work.

Order
Grid Algebra uses a natural sense of order. Students can often recall visually and kinaesthetically the order of movements carried out and this helps them read order into the resulting expression created through those movements. You may be surprised at how students learn to read quite complex mathematical expressions.

Order of movements spatially relates to *order of operations* mathematically.

When expressions are entered into a cell through the Expression Calculator, they are entered in the order of operations rather than a left-to-right order. This helps reinforce a sense of order of operations within expressions.

Inverse
Grid Algebra uses a natural sense of inverse whereby if I move a certain distance to the right and want to get back to where I started then I know I have to move the same distance but to the left. So if I move two to the right in row 1 (adding two) then to get back to where I started I move the same but to the left (subtracting two). Likewise the inverse of moving down (say multiplying by 4) will involve the same but going up (dividing by 4). There is also a natural sense that when doing a combination of moves, say moving to the right and then moving down (adding followed by multiplying), the inverse journey would be moving up followed by moving to the left (dividing followed by subtracting).

Inverse movements spatially relate to *inverse operations* mathematically.

Letters
Since attention is placed on movements and journeys around the grid, which number was actually in the start cell of a journey becomes less significant. So students often have little difficulty with re-naming that cell with a letter. This is done through dragging any letter from the Letter Box into that cell. In this way journeys result in algebraic expressions being produced which involve that letter. This helps the transition from arithmetic to algebraic work be relatively seamless.

Dave Hewitt, 2007.