TSM Conference 2003

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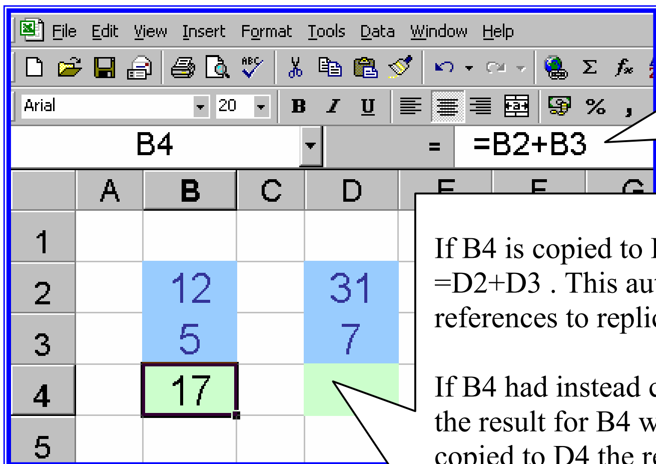
Key Problem Solving Techniques:

A small collection of simple techniques can, with a little imagination, open up large areas of mathematics in interesting ways.

Useful Excel techniques include:

- **Absolute cell references** – the normal state for the cell references in a formula is to be a *relative* reference. This means that when that formula is copied to a new position it builds a new formula with the cells references that relate to the new position rather than returning to the cell references used in the original formula. For example:



This is the formula for the cell B4. It adds together the two cell values immediately above.

If B4 is copied to D4, D4 will receive the formula =D2+D3 . This automatically contains the correct cell references to replicate the effect that B2+B3 had in B4.

If B4 had instead contained the formula B2+B3, the result for B4 would remain the same but when copied to D4 the references would not adjust automatically. D4 would get exactly the same formula as B4 not its own formula to achieve the same effect.

- **The INT function** drops the fraction part of a number and just returns the integer part. INT(4.1623) would return 4 , INT(4.8623) would also return 4. INT has many uses, but the two covered in this workshop are

    - Getting at the individual digits in a number.
      (see technique note: Getting at Digits using the INT function)

    - Creating random numbers in a given range.

  Remember in Excel = is needed at the start of a formula to be calculated, otherwise Excel assumes that INT is just a word you wanted to type.

- **The ABS function** returns the absolute value: ABS(20 – 50) would return 30 not – 30.

- **The MOD function** returns the remainder when one value is divided by another. MOD(15,2) would return 1 , MOD(15,3) would return 0.

  The MOD (modulo) function is particularly valuable when exploring factors, multiples, and remainders.

- **Conditional Formatting** – Excel has the ability to change the appearance of individual cells based on the cell's contents at that time. (see technique note: Conditional Formatting)

  Conditional Formatting is particularly powerful if values of interest are hard to identify, just by inspection. The Beads investigation introduced in the first session is a good example. Conditional formatting was set to ensure that remainders of 1 were conspicuous.

- **Working with two independent variables**
  (see note: To produce a table of values for a function of two independent variables)

  The Pythagorean Triples investigation makes interesting use of this technique in conjunction with *conditional formatting.*

- **Generating random numbers**

  The command **:** **=RAND()** will produce a random value between 0 and 1.

  So **=10*RAND()** will produce a random value between 0 and 10

  and **=INT(10*RAND())** will produce a random integer between 0 and 9

  =1+INT(10*RAND()) will produce a random integer between 1 and 10

  It is possible, and worth a try, that your school PC has the RANDBETWEEN function installed. This is not set up as the default installation of Excel.

=RANDBETWEEN(10,30) would produce a random integer in the 10 to 30 range.

The Help index in Excel will give guidance about extras functions which can be installed.

It is also a useful problem solving task for pupils, just once, to generate the required random value range using only the basic functions as building blocks.

Note: NRICH web site

The following web page has a large number of working Excel files demonstrating and extending these notes on paper:

http://nrich.maths.org/mathsf/journalf/rb_excel_archive.html

A new Excel@Mathematics item appears each month on the NRICH site (under features).

Beads:

When an unspecified quantity of beads is divided into two equal piles one bead is left over. When the same heap is divided into three, four, five, or six, equal piles, that remainder of one persists. With this information can you say how many beads there were?
Is a solution even possible? Is there more than one solution? Was the result predictable?

Extend the enquiry by investigating other remainders, other divisors, or even other rules for distribution.

Approaching a solution (off-screen planning is essential), one layout could be:

Column one: the number of beads. The first row has one, the next two, and so on.
Column two: the remainder after a division by two
Column three: the remainder after a division by three
Until all the required calculations are displaying their results .
Solutions occur wherever a row contains all ones.
Conditional formatting has been applied to automatically highlight cells that hold a value of 1, but this is not necessary.

It is now immediately apparent that one solution is to have 61 beads, while scrolling down further reveals more solutions at 121, 181 and so on at intervals of 60.
Spared the tedious individual calculation we are free to ask: "why intervals of 60"?

We might look for a relationship between this interval of 60 and the divisors, 2, 3, 4, 5, 6, and notice that 60 is the lowest common multiple of all these divisors.
Is the lowest common multiple relevant to the problem?

This demonstration compresses the problem solving process from a couple of hours to a few minutes. The problem is the focus, not the spreadsheet, which has merely produced helpful numerical results. The mathematics happened when we were trying to understand the problem, trying out some numbers manually, thinking how a spreadsheet might be useful, planning what was needed and how to organise the calculations in columns, interpreting the results, making connections, trying to justify our insights to ourselves and to others.

Investigation techniques applied to two examples *More Beads* and *Ring on a String*

More Beads

A heap of beads was shared out between three people by a professional bead sharer. The first person got one third, but not before the bead sharer had taken one bead as commission. The second person had one third of what remained but again, after commission of one bead had been taken by the bead sharer. The third person also received a one third portion of what remains but, as before, after a one bead commission had been taken.
Once all three people had received beads the bead sharer took a final commission of one bead and shared the remaining beads equally between the three people. There were no beads left over and beads were not split into fractions. How many beads were there at the start?

As always, start by trying out some numbers to get a feel for the procedure you will be investigating. Next plan out on paper what calculation each of the columns will contain and also what range of numbers you will use initially.
It helps to have several columns taking the procedure in small steps rather than trying to leap to the answer in one complex calculation.

Here's a possible plan:

- Column 1 – Number of beads. Maybe start at 1 and perhaps run down to around 200 initially.
- Column 2 – The number of beads after the bead sharer's first commission of 1 bead
- Column 3 – One third of the beads
- Column 4 – The remainder when this third is removed
- Column 5 – The beads after the second commission of 1 bead
- Column 6 – One third of the beads at this point
- Column 7 – The remainder when this third is removed
- Column 8 – The beads after the third commission of 1 bead
- Column 9 - One third of the beads at this point
- Column 10 - The remainder when this third is removed
- Column 11 – The beads after the final 1 bead commission
- Column 12 - The final division into three equal amounts.

If the numbers right through to the end of the row remain integer, then the start number of beads was a solution to the problem.
The row gets quite long. Trying to look at both the beginning and the end is frustrating. It can be helpful to create a new row at or near the beginning and make this show (repeat) the value at the end.

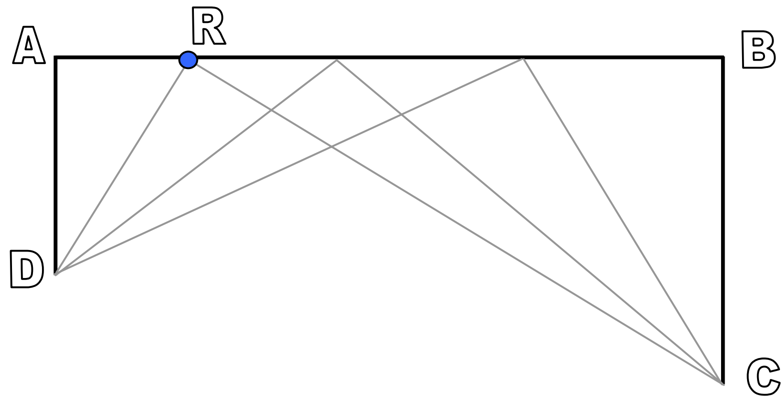
Good investigation practice, with or without a spreadsheet.
Interesting results often come from procedures that are too complex to be obvious.
We need to teach pupils to strip back to the simplest version of the procedure, and develop forward to the actual context in small steps.

So in this investigation that would mean just two people and no professional bead sharer taking a commission. Then three people. Perhaps four people if a pattern is emerging. Then look at the effect when the commission payment operates.

A key investigation skill : working backwards
The whole procedure could be represented on the spreadsheet in reverse.
Starting with the quantity in the final share round. Successive columns would show the number of beads prior to that stage until the final column which would show the number of beads required at the start.

Ring on a String (see the pupil worksheet - *Ring on a String*)

Some start value choices have to be made, then remade as the investigation develops.



If AB was 100, AD 30 and BC 50. There is now only one variable, the position of R along AB.
If the first column is the distance of R from A starting at 1 and increasing by 1 until 100 is reached.
The distance DR is the hypotenuse of a known right-angled triangle, and can be calculated with Pythagoras' Theorem. Similarly for CR. The ring will settle wherever the total DRC is least. So the final column is the sum of the lengths DR and CR. Casting an eye down this column for the minimum value is all that is required to bet a good first result. (An approximate result because the possible positions of R were taken at intervals of 1)
Next the position of D or C can be altered in a systematic way until sufficient results are produced for any pattern to become clear. Then the key question: "Why that pattern?"

The INT function allows digits within a number to be isolated. Many investigation tasks rest on a property of the digits. The technique help sheet: Getting at Digits – Using the INT function , explains what to do and suggests an investigation task called *Casting Out Nines* which is based on the Digit Sum.

The next two investigations, *Pythagorean Triples* and *The Sum of Consecutive Numbers*, use Excel to support group discussion. The Excel display provides instant interactive response as pupils form and test out their ideas, but at Key Stage 3 only very able pupils will want to explore the construction method.

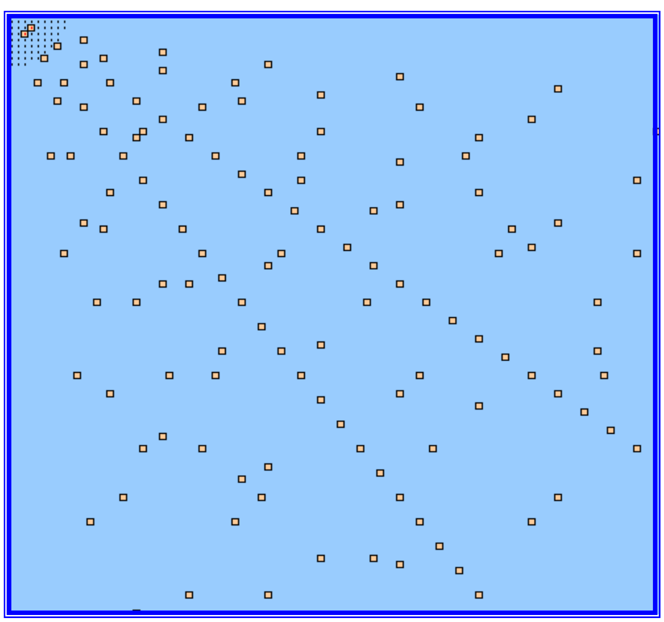
Pythagorean Triples

Find sets of three numbers so that the sum of the squares of the first two is equal to the square of the third, as in (3, 4, 5) or (5, 12, 13).
Using the Excel technique for problems with two independent variables create a table of all possible first and second number combinations to see which of these provide integer results for the square root of the sum of their squares.

The formula for the first cell is =SQRT(B1^2+A2^2) which is then copied across the chosen range. Then look for cells in which this formula returns integer values. Conditional formatting helps identify the values of interest. The colour change occurs when a cell value is exactly equal to its integer part.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	1	1.4	2.2	3.2	4.1	5.1	6.1	7.1	8.1	9.1	10	11	12	13	14	15	16	17
3	2	2.2	2.8	3.6	4.5	5.4	6.3	7.3	8.2	9.2	10	11	12	13	14	15	16	17
4	3	3.2	3.6	4.2	5	5.8	6.7	7.6	8.5	9.5	10	11	12	13	14	15	16	17
5	4	4.1	4.5	5	5.7	6.4	7.2	8.1	8.9	9.8	11	12	13	14	15	16	16	17
6	5	5.1	5.4	5.8	6.4	7.1	7.8	8.6	9.4	10	11	12	13	14	15	16	17	18
7	6	6.1	6.3	6.7	7.2	7.8	8.5	9.2	10	11	12	13	13	14	15	16	17	18
8	7	7.1	7.3	7.6	8.1	8.6	9.2	9.9	11	11	12	13	14	15	16	17	17	18
9	8	8.1	8.2	8.5	8.9	9.4	10	11	11	12	13	14	14	15	16	17	18	19
10	9	9.1	9.2	9.5	9.8	10	11	11	12	13	13	14	15	16	17	17	18	19
11	10	10	10	10	11	11	12	12	13	13	14	15	16	16	17	18	19	20
12	11	11	11	11	12	12	13	13	14	14	15	16	16	17	18	19	19	20
13	12	12	12	12	13	13	13	14	14	15	16	16	17	18	18	19	20	21
14	13	13	13	13	14	14	14	15	15	16	16	17	18	18	19	20	21	21

The red type on a tan background are genuine integers, and are the result of the conditional formatting being activated. The other integers in the sheet are caused by Excel rounding the display value to 2 digits even though the held value may have many decimal places, as increasing the width of a column would reveal.
With a Zoom view of 10% some patterns become obvious. Pupils should be encouraged to account for the patterns they observe. (reflection in the diagonal and enlargement or similar triangles are the most obvious patterns)



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The Sum of Consecutive Numbers

The pupil sheets *The Sum of Consecutive Numbers* cover the main activity. Excel is used here, as a ready made file, to provides an excellent interactive presentation to support class discussion as pupils form and test their own ideas.

The top row is the start number for the consecutive number sequence.
On the left , the first column gives the sequence length.
For example: 4 + 5 + 6 + 7 + 8 + 9 starts at 4 and run on for 6 terms, to make a total of 39.

15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
2	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
4	10	14	18	22	26	30	34	38	42	46	50	54	58	62	66	70	74	78
5	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	21	27	33	39	45	51	57	63	69	75	81	87	93	99	105	111	117	123
7	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147
8	36	44	52	60	68	76	84	92	100	108	116	124	132	140	148	156	164	172
9	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198
10	55	65	75	85	95	105	115	125	135	145	155	165	175	185	195	205	215	225
11	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253
12	78	90	102	114	126	138	150	162	174	186	198	210	222	234	246	258	270	282
13	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312
14	105	119	133	147	161	175	189	203	217	231	245	259	273	287	301	315	329	343

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Conditional Formatting Notes

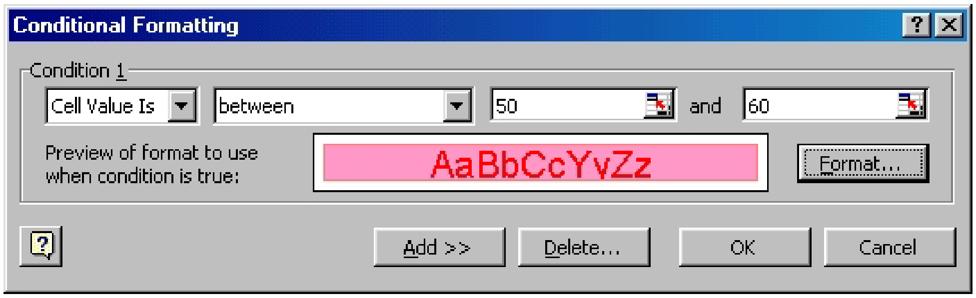
See the web page: www.nrich.maths.org/mathsf/journalf/jul03/excel.html
On the Excel File called "Conditional Formatting Demonstration", sliders control C2 and C9, spinners control C13 and C14.

Conditional Formatting is a setting applied to a cell which causes the appearance of the cell to change if a specified condition occurs.

There are two basic types of conditional formatting. One where the condition involves the value in that cell, the other where the condition is based on values from other cells around the sheet. The demonstration Excel file gives an example of each.

If C2 's value goes into the 50 - 60 range you'll see the cell's formatting change.

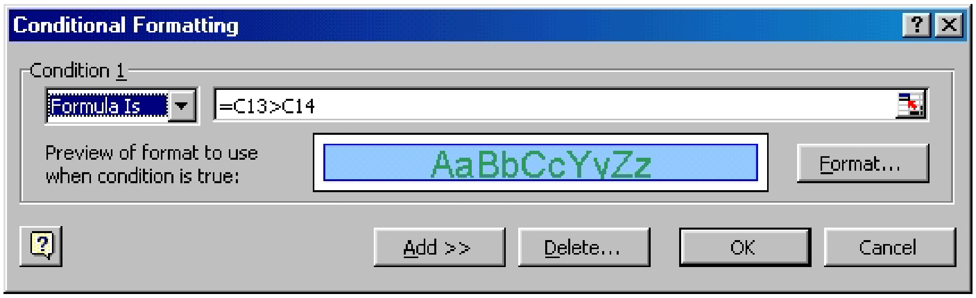
Click on C2, then choose Conditional Formatting on the Format menu to see the settings used.



Clicking the " Format . . . " button allows different colours to be selected.

The next slider controls C9.

C9 also has Conditional Formatting applied, but this time the condition is not dependent on C9 's own value.
C9 can be made to change its appearance based on values elsewhere in the sheet.
In this case, the values in C13 and C14. C9 's own value has no influence.
Here's the setting:



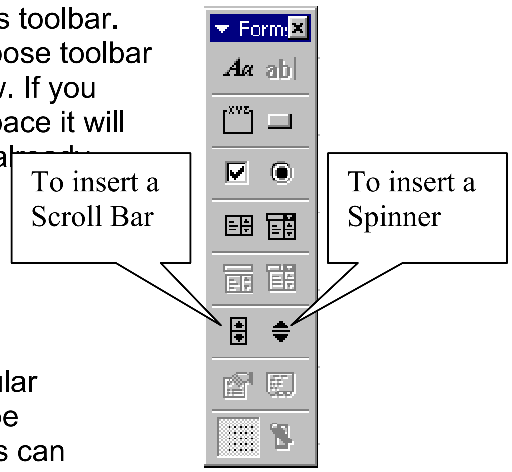
Notice that the first field in the dialogue box is set to " Formula Is ", and that, in the second field, the condition has to start with an " = " sign.
Excel needs the " = " to know that what follows is a formula, to calculate or test.

So, with this setting, if C13 > C14 the alternative formatting for C9 is triggered.

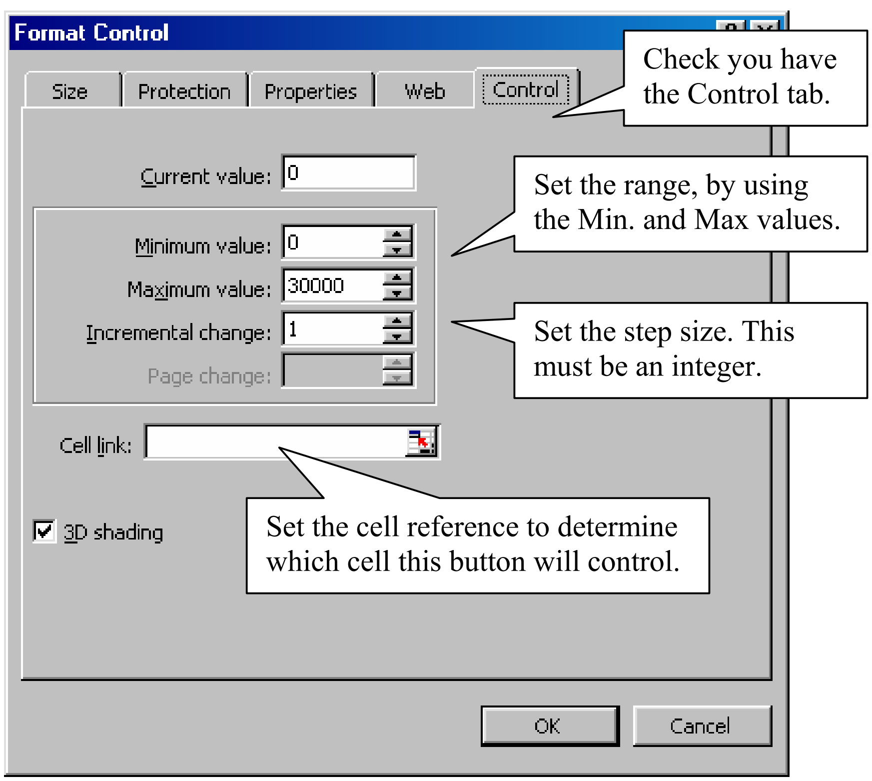
Inserting an Increment Button (Spinner) or Scroll Bar

to control the values in a cell

- The cell values are usually entered on the keyboard, but sometimes it is desirable to keep attention completely on the spreadsheet and not to use the keyboard at all for entering or changing cell values, e.g. with an interactive whiteboard. This can be achieved using increment buttons. There are two types available, Excel calls them Spinners and Scroll Bars.
- The Spinner is a two part button made formed from an up arrow and a down arrow. The control is set on the Spinner so that the values stay within a chosen range and only change in steps of a specified size. The scroll bar is the same but with a slider between the two arrows.
- Spinners and Scroll bars are found on the Forms toolbar. To turn on any toolbar go to the View menu, choose toolbar and select the toolbars you wish to have on view. If you drag a toolbar to the top or bottom of the workspace it will become absorbed amongst the toolbar buttons already there.



- Chose the tool (spinner or scroll bar) from the toolbar, then just click and drag out the rectangular outline for the button you wish to create. It can be any size. Note: the size and other characteristics can be changed at any time.
- After you have a button on the workspace the next task is to connect it to a cell. Right-click on the button, choose Format Control, and complete the dialogue box.



To produce a table of values for:
A function of two independent variables

	A	B	C	D	E
2			5	7	9
3		2	10	14	18
4		4	20	28	36
5		6	30	42	54

The cell C3 contains the formula: = C$2 * $B3

* means multiply, C2 is the factor from the top row and B3 is the factor from the left hand column. The dollar sign, $, ensures that when the formula is copied from C3 across the whole C3:E5 range the first factor will continue to be taken from row 2 and the second factor will still be taken from column B.

The keystrokes for the copy manoeuvre are:

- select **C3** (just click on it)
- choose **Copy** from the **Edit** menu
- highlight (click and drag) the **C3:E5** range
- choose **Paste** from the **Edit** menu

The formula from C3 is copied automatically to all cells in the C3:E5 range. The formula is adjusted for each new position except where a $ sign was placed to show that this automatic assistance is unwanted.

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