Association of Teachers of Mathematics

Integrating Computers - Ronnie Goldstein

Micromath

Mathematics and IT - a pupil's entitlement (CITS, 1995) describes six major opportunities which IT offers to those learning mathematics: learning from feedback, observing patterns, seeing connections, working with dynamic images, exploring data and teaching the computer. A few years ago a similar document might have been structured by a consideration of major software applications such as spreadsheets, databases, dynamic geometry, graphic calculators and the Logo programming language. When any technology is unfamiliar we have to start by looking at it out of context, considering what it can do rather than what we can do with it. But, as time moves on, we begin to understand that IT must be integrated into the curriculum if it is to have any purpose in schools. This is not a new idea but, never-the-less, current practice suggests that it is not inappropriate to reiterate the point in this article about the place of IT in the learning of mathematics.

The computer is a tool and as such it must usually be secondary to the task in hand, a servant of the curriculum. In most secondary schools today there are not yet enough computers for them to be accommodated in all teaching rooms and also in computer rooms where all children can gain access. For various reasons, computer rooms have taken priority and, inevitably, students do not always appreciate the connection between their computer activities and the curriculum. When computers are more prevalent, we will all benefit in the way that owners of laptops or graphic calculators benefit today. The students will be using a personal, flexible tool, which is always accessible to them and yet never dominates their attention. It will be used only when there is an appropriate context and when it is no longer needed, it will be put away.

A number of classroom activities where the computer has an important role are described in this article. In each case the benefit of additional work away from the computer is stressed. The following section, *Explaining patterns*, emphasises the need for explanation and justification in mathematics and *One amongst many* contains descriptions of work with the computer where activities with other resources are complementary.

Explaining patterns

When exploring mathematical problems, the speed of computers and calculators enables students to produce many examples. This enables them to observe patterns so they can readily make generalisations and it is clear that, without the computer, the students would be disadvantaged. However, the computer may not be enough. To see a pattern is important but in mathematics we want the students to understand it and to be able to explain it.

Students using a computer might easily 'notice' the following relationship: the square of a number is always 1 more than the product of the two numbers on either side of the original number.

For example, 10 x 10 is one more than 11 x 9, 4.7 x 4.7 is one more than 5.7 x 3.7 and this will be true for any starting number. The spreadsheet below illustrates the relationship for some very awkward starting numbers.

Number	Square	(Number+1) x (Number-1)
-6.27	39.3129	38.3129
-4.88	23.8144	22.8144
-3.49	12.1801	11.1801
-2.1	4.41	3.41
-0.71	0.5041	-0.4959
0.68	0.4624	-0.5376
2.07	4.2849	3.2849
3.46	11.9716	10.9716
4.85	23.5225	22.5225
6.24	38.9376	37.9376

But is the result *always* true? The table of results is fairly convincing and most students will be satisfied with the truth of the general statement. In order to explain the result and really understand it, however, it is necessary to leave the computer and, if the students can cope with the formal algebra, prove that:

$x^2 - 1 = (x - 1)(x + 1)$.

Alternatively, a geometric model might be used as in figure 1. The square with the corner missing represents the left-hand side of the identity and, when one strip is moved, a rectangle representing the right-hand side is formed.

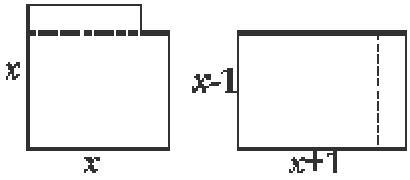

Figure 1

The explanations above are not only important because they start to satisfy the pure mathematician's desire to justify the result universally, for all numbers. (There are infinitely many numbers and a spreadsheet can never test them all.) The explanations also get closer to the heart of the mathematics being studied. The computer has allowed the students to explore, with all the freedom to be wrong. Indeed, it is through exploration with the computer that the result may first have been suggested. But it is when a student is able to argue and convince someone else, that she has understood some new mathematics.

The computer sometimes encourages us to work in ways which do not necessarily lead to analytical solutions. The machine is a workhorse, which can generate so much data so quickly that we may be tempted simply to spot patterns without any analysis of the situation under consideration (Goldstein and Pratt, 1994). Another example of spotting patterns without explanation of the results might occur when dynamic geometry software (CITS, 1996) is used for measurement.

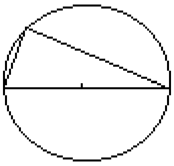

Figure 2

For example, if a student draws a circle (Figure 2), places a point on the circumference, joins the point to the opposite ends of a diameter and then drags the point around the circumference, on measuring the angle at the point she will be bound to notice that it is a constant 90°. There is no guarantee, however, that she will understand why.

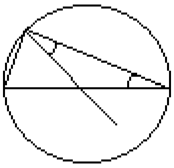

Figure 3

To understand the result the student needs to leave the computer alone and show how logic demands that the angle at the circumference is equal to the sum of the other two angles that comprise the original triangle. (The new line passes through the centre of the circle and the marked angles are equal because the triangle is isosceles).

The computer offers many powerful opportunities for generalisation based on experimental evidence and the process of exploration leading to such generalisations can be a vital component of students' learning. In some cases the exploration may lead naturally to the justification but, where this does not happen, the teacher may have to intervene. Indeed, a teacher needs to try to find opportunities to ask students why a result is true whenever they stumble across one.

One amongst many

The activities in this section are about locus and tiling; in each case, the computer provides only one approach amongst many that might be valuable.

LogoMetric (ATM, 1994) is a Logo microworld, which might be used to plot points that are twice as far from one of the small circles as they are from the other. One of the turtles is used to 'carry' the two important lengths as it is dragged around the screen with the mouse. When one of the numbers is twice the other, the turtle's position is stamped (with a cross) by clicking the mouse.

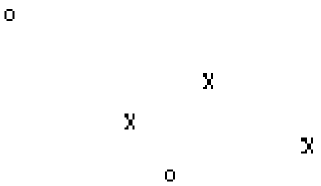

Figure 4

The crosses are all twice as far from one of the circles as they are from the other.

The computer activity can be vital for an understanding and exploration of loci and the students could quite easily continue to plot many more points where the relationship in question holds true. But there is more to this activity than can be achieved by pressing a button on a keyboard hundreds of times and having the computer supply all the answers. The teacher might want to ask her class to try to visualise the shape of a locus:

"Close your eyes. In your mind can you see two small circles? Put a cross near one of the circles and move it slowly towards the other one until it is the same distance from each. Now move it further, so that it is twice as far from . . ."

Learning to visualise in the mind is an important aspect of mathematics, which is unlikely to happen while students are only using computers.

Another approach to the same topic might be to work with the class in the hall or playground and to ask everyone to move so that they are twice as far from one chair as they are from another. This forces every student to be central and active. Using counters on paper provides a better picture of the final result; this is a third activity which also complements the work with the computer. Each of these activities has its own strengths and, time permitting, each would add to the learners' experience and understanding of loci.

When computers are used to serve the curriculum, the relative merits of the technology and other resources can be considered. If, for example a student is using software to design tiling patterns, she will undoubtedly benefit from the speed and accuracy of the micro; she will be able to create several different shapes and test them all to see if they form tiling patterns. But, on the other hand, she may need to do some things more slowly, so that she has time to understand the rationale of her approach. If she has never used scissors and card, or at least plastic shapes, to make tiling patterns, she may never reach a full understanding of the design of the pattern which the computer achieves so quickly. This does not imply, however, that the computer can only be used when the student fully appreciates all the concepts. Forming partially understood patterns quickly and easily can motivate students to understand what is happening.

Work with scissors and card should be happening alongside the computer. Perhaps the most important message we can learn from all of this is the obvious one: don't let the students try to learn anything by using one approach alone. Learners need a variety of contexts in which to experience new concepts. The computer can be exciting and worthwhile but it is one road among many to a full conceptual understanding.

In discussing the relative merits of computers and other resources there is a danger that I have implied that the results for the student are the same and that the same mathematical content is being addressed. Allow me to consider tiling activities in a little more depth so that I can illustrate how false this can be. *Put and take* is an idea well-known in the primary school.

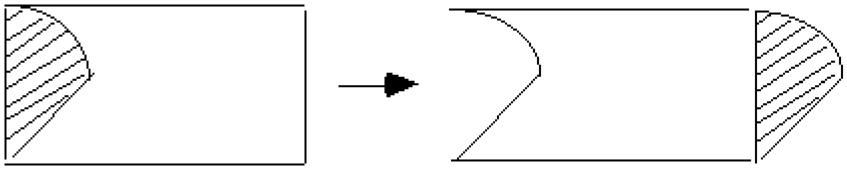

Figure 5

The essence of the activity, when it is done with card and scissors, is to cut a small piece from one side of a tile and stick it on the opposite side. This produces a new, less regular shape and, provided the original shape can be used as a tile, so too can the new one.

At first glance *Reptile* (Kudlian Soft, 1994) might appear to offer a similar experience, but there are important differences. When children work with card and scissors, the emphasis is on the piece of card to be re-positioned and the topic of area can be addressed. With *Reptile*, however, the students have to manipulate the edges of the tile. The same result is achieved by distorting one of these edges and then copying it, so that two of the tile's edges are identical. At no point is the student likely to be considering the area of the tile – the emphasis on the identical edges.

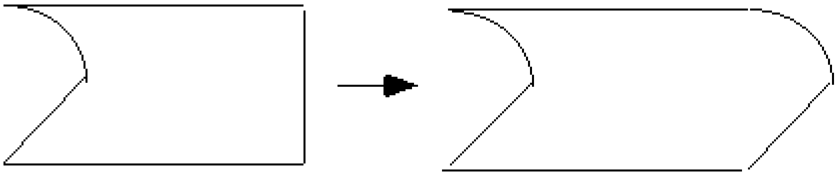

Figure 6

It has often been observed that the medium of any activity fundamentally affects the message and it is important to be aware that different approaches in the classroom will not produce identical results. Hopefully, they will be complementary.

The future

It may be that some smaller pieces of educational software can be used profitably on their own, without reference to other classroom activities, but as soon as the computer is used in more significant ways it becomes important that the work happens alongside other related activities. This article has shown how work with the computer often needs to sit alongside other activities but, in the main, even these examples have been driven by the computer. In ten years' time I hope to be able to write an article about the secondary school classroom where much of the work starts well away from a machine. The stimulus for the work will be the mathematics curriculum, and the computer (which may be used quite often) will not always be at the centre of the teacher's thinking.

This is not easily achieved today, while computers are not available everywhere and at all times. In the future there will be further issues to consider; when computers are fully integrated with other mathematical activities, it will be important for pupils to learn to make their own decisions about their use (NCET, 1994). They will need to decide whether or not it is appropriate to use a machine for a particular mathematical task or activity. If it is appropriate, which software should be deployed? Should the machine be used at the start of the activity or after some initial research? And, perhaps one of the most difficult questions, when should the machine be switched off?

Ronnie Goldstein works at NCET in Coventry.

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