Re Mathematics Teaching NT187 June 2004

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It was good to read Paola Iannone's "Did I tell you my Pythagoras horror story?" (MT **187**, 13-16).

The substance of her method was to ask lecturers to discuss unsatisfactory student responses. The question that the students had been asked was

> 'Let $x \in \mathbf{R}$ have the properties that $x \geq 0$ and $\forall n \in \mathbf{N}$, $x < 1/n$. What is x?'.

At no point in the discussion did lecturers suggest that students might have responded differently had the wording of the question been revised. Even to write the inequalities in the question in the form $0 \leq x < 1/n$, would have made the squeeze in the question more obvious. And there are other variations which might usefully have been explored. The quantifier '$\forall$' contributes a gratuitous mystery. In ordinary sixth form usage, x is a variable; here one grapples with the question by thinking of x as constant.

The question might have been broken into two parts.
(i) If a constant c is given, and $-x < c < x$ for all positive numbers x, does c have to be 0?
(ii) If a constant c is given, and $-1/n < c < 1/n$ for all natural numbers n, does c have to be 0?
And then further variants, first with $\leq$ replacing $<$, and later posing the question for non-negative numbers, may clarify the situation for the student. There is a lot here to unpack. Solving problems is close to the heart of learning, so getting the problems expressed in the form which generates the most learning is worth doing.

Lecturer T's concern about infinitesimals should emerge with different answers to parts (i) and (ii), since Archimedean order is critical for (ii), but irrelevant for (i).

Lecturer S was concerned for students flummoxed about the proof. But the question here is resolved only by contradicting the supposition that 0 is the only solution. Proof by contradiction is not an A level skill, and deserves being taught in its own right. And I don't mean by having lessons in logic. When students have tried and failed to put together a constructive proof, in despair they may suppose that the question is wrong and explore the consequences. At that point they may be able to generate a proof by contradiction.

The particular question offered is of great moment. It can be used as a basis for developing the notion of limit.

Lecturer P wisely noted that the languages of sixth form maths and that of undergraduate pure maths are different. But he made no comment on how to build a bridge between the two languages. The students are being immersed in the lecturers' language. Learning to set questions so that students grapple with the mathematics, rather than remaining fuddled by expression and notation, is a skill for lecturers to learn. There is a famous quotation from Ausubel (1968):

> If I had to reduce all of educational psychology to just one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this, and teach him accordingly.

Iannone's article makes it clear that Ausubel needs to be heard again.

Thank you for publishing an article about the teaching and learning of undergraduate maths.

Bob Burn