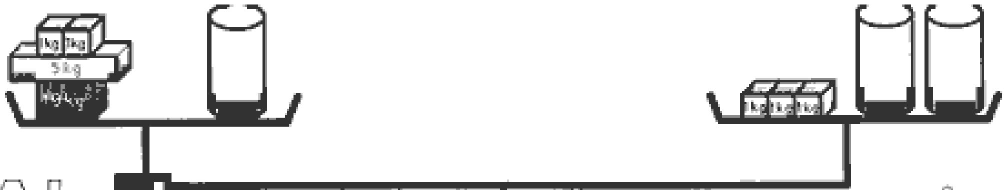
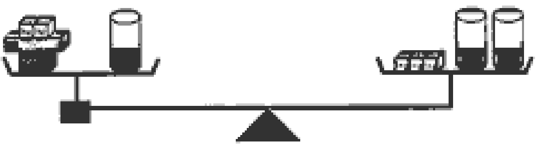


Algebra: Rebalancing the equation

James Aczel and Dominic Tilley

How can technology help more students to appreciate the power, beauty and challenge of algebra? Past issues of *Micromath* have provided many useful activities involving calculators, LOGO, Derive and Excel. We wondered though, whether rather than using the technology to encourage students to make sense of symbolism, we could make the choice of an algebraic strategy more attractive. We worked with a Year 10 class that had been taught much of the GCSE intermediate syllabus, but contained many who struggled with algebra and seemed to see it as a pointless ritual.

Balance Puzzles



The balance puzzles commonly found in KS3 textbooks to teach the idea of solving equations by simplification provided a starting point.

We noted that many children cope well with these because they are readily accessible to informal strategies such as trial-and-error and covering-up. However, there are difficulties:

1. The balance metaphor cannot cope easily with negative signs and negative solutions.

2. With physical balances, spontaneous use of a cancellation strategy rather than informal strategies has been found to be rare.

3. The idea that symbolism provides a useful shorthand is easily frustrated by the need to stick to standard conventions that avoid confusion. The question "Solve $2x + 4 = 10$" elicited the response "$2x + 4 = 10 - 4 = 6 \div 2 = 3$" from one student.

4. For many children, the idea that formal operations on equations are a way of making the problem easier is frankly laughable because they know that the slightest error in arithmetic, notation or procedure can render the whole exercise fruitless and perhaps a little scary.

5. Research has pointed to dangers in perceiving a letter as an object rather than as an unknown – it is not the barrel that is abbreviated by the letter b in $17 + b = 3 + 2b$, but the barrel's weight. Letters are also used here as unknowns, and not variables, which some researchers have suggested is too limiting a way to start algebra.

Promoting simplification

We presented the class with a program that generated a series of balance puzzles, on the computer, that only get harder when the students can solve them. Rebecca and Nicola, for example, solved the first few with little difficulty until they reached a puzzle with barrels and weights on both sides. Faced with a picture showing 19kg and 4 barrels on one side, and 12kg and 5 barrels on the other, they noticed a button on the screen labelled "Take off barrels". They clicked on this, and were given the prompt "How many barrels?". They decided to take off 1 barrel, and saw that one barrel had been taken off each side. They decided to take off 1 again, and then another. At this point (19kg and 1 barrel on one side, and 12kg and 2 barrels on the other) we expected them to take off the final barrel. But they didn't. Stuart passed by and suggested taking off the final barrel. "Yeah", said Rebecca to Nicola in confident tones, "take off one barrel.". There was a pause, and then she asked Stuart, "What... have you done this one before?". But when she saw the new picture (19kg on one side, 12kg and 1 barrel on the other) she exclaimed "Ah that's obvious now", because it was now just like the earlier puzzles. Soon they were taking off the barrels in one go (rather than singly). As in a computer game, they were promoted up a level and started to amass a score that they eagerly compared with their friends' scores.

The unthreatening introduction of symbolic notation

As the puzzles got harder, they started using the "Take off weights" button to deal with larger numbers, and the [/] key to deal with fractional answers. The two "Take off" buttons became a single [−] button, and they quickly worked out how to use it to subtract weights and barrels (e.g. [−] to take off 3kg, but [−] $3b$ to take off 3 barrels). On promotion to the next level, they found that the concrete, multi-coloured pictures had been replaced by symbolic notation. Far from confusing them, they continued with their simplification strategy and in fact were even quicker than with the pictures because they no longer had to count barrels.

Does this astonishing transfer from concrete to symbolic hold good for those without prior algebraic experience? Evidence from trials of the program suggest that even Year 6 children found the jump from pictures to symbols perfectly natural, and that they enjoyed the whole experience.

Feedback on solution processes

Now that the balancing context had been completely left behind, negative signs and negative solutions became possible. When equations simplified down to, say, $39 + 3b = 0$, several students commented that this doesn't make sense – even though they are familiar with negative numbers – and they then asked for help. It doesn't "make sense", of course, within the confines of the balance metaphor, in which b has to be positive. The question is: how can one enkindle a questioning of the limits of that metaphor? We found that saying things like "Well, is there anything you could do now?" or "Can you still take things off?" encouraged them to subtract either 39 or $3b$ to leave $3b = -39$ or $39 = -3b$. At this point they were then often happy to enter a negative answer because (saying, e.g., "If the minus wasn't there you'd do 3 into 39, so do 3 into -39").

Enabling such exploration of algebraic form is, of course, one of the educational advantages of this software environment, as demonstrated by the following program log, in which the [+] button is used:

#	Action	Resulting Equation	Time(s)
		$-22 - 14b = -12 + 4b$	
1	Subtract 12	$-34 - 14b = -24 + 4b$	6
2	Subtract 4b	$-34 - 18b = -24$	10
3	Add 24	$-10 - 18b = -0$	11
4	Add 10	$18b = -10$	11
5	Guess -18/10		14
6	Guess 10/-18	Correct	9

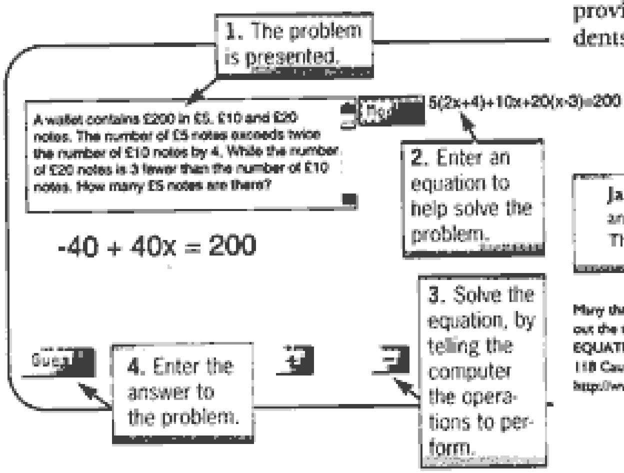
Note how the feedback to Action 1 allowed them to debug their strategy. Nicola said "You should've plussed 12.". Rebecca isn't convinced, but tries it anyway and the feedback convinces her. The program logs then show a gradual construction of strategic theories for dealing with negative signs in equations.

With pencil-and-paper, students not only have to choose the operations to perform, but they also have to carry them out correctly. With this program – rather prosaically named "EQUATION" – although the student chooses the operation to perform, EQUATION executes it. This means firstly that attention can be devoted to simplification decisions, without worrying about arithmetic; and secondly that students see the effect of an operation instantly, thus preventing pages and pages of error-strewn workings. Rebecca and Nicola seemed well aware that choosing an unhelpful operation is not fatal to the solution process, because the solution is unaffected. Of course a degree of feedback can be provided by a teacher, a textbook or a fellow student. The computer has the advantage of speed, which makes it less likely that the student will lose the thread. Students therefore have the opportunity to experiment with algebra, to try hunches and make mistakes that do not lead to having to start from the beginning. The evident satisfaction derived from this success kept the whole class on task for an hour's lesson.

At this point we raise two points. Firstly, we felt that Rebecca and Nicola needed more practice in equations with negative signs, to consolidate the variety of possible permutations. Secondly the transfer from computer-based solution to pencil-and-paper is non-trivial, and it would have been a useful moment for Rebecca and Nicola to start to try to solve equations without the computer, perhaps checking their strategies using the computer. Professional judgement is therefore required to decide, for the individual learner, what interventions might be appropriate.

The opportunity to formulate equations to solve word problems

Now that the class could successfully solve simple linear equations, could they use their knowledge to help solve word problems? The first word problems they were given were simple descriptions of balance pictures, and it took little trouble for them to formulate an algebraic model of the situation. Using an equation was optional, and some students reverted to trial-and-improvement, or even guessing blindly what operations might be applied to the numbers given in the question to get the right answer! But the next level of modelling problems involved puzzles with negative signs; the next involved combining ratios; and the final one required the use of expressions; and the "Model" button came into its own. The amazing thing was not so much that students were able to formulate their own equations, but that once the equation appeared on the screen (simplified into the standard $ax + b = cx + d$ form) students said things like "Ah, now I can do it!" and "It's easy now!". In other words, the equation had become for them a powerful problem-solving tool which they were confident about using.



In retrospect, it would have been better to have graduated the difficulty of the modelling puzzles a little more – being asked to find a quantity that was not the obvious choice to be represented by a letter (see above) was a bigger hurdle than we anticipated. However, appropriate teacher intervention would be able to deal with this.

Conclusion

The year 10 class using EQUATION for two lessons improved significantly on a written algebra test. However, the software is certainly no panacea: there are problems for which it helps, problems for which it does not help, and problems where the effect is unclear. Moreover, it does not replace the need for activities involving identities, variables, spreadsheets or programming – it leaves until later appreciation of algebra as a language for expressing generality. But the students clearly improved their equation-solving skills; they showed an increased concern to use algebraic strategies; and they seemed to understand the role of algebraic representation better. Also, the objections that might be applied to the use of a physical balance or a balance picture in a textbook do not apply to the balances in the EQUATION.

We think that the reason why the software works is that it makes a simplification strategy attractive, it introduces algebraic ideas gradually, it gives instant feedback, and it provides a purpose for standard algebraic representation through situational problems. The best outcome, though, is that everyone really enjoyed using algebra! The use of such an interactive, game-like, computer-based balance model could provide the key to unlock algebra for many students.

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EQUATION is available for Windows 3.1 and Windows 95 from Equation Software, 118 Causeway, Banbury, Oxon., OX16 8SQ.
http://www.webcom.com/dream/equation/