Association of Teachers of Mathematics

Calculators: Time to Take Stock - Tony Gardiner

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Calculators and computers are part of the tapestry of modern life. This fact no-one would dispute. However, in certain (largely English-speaking) countries it has been fashionable to conclude, without clear supporting arguments, that these changes have profound implications for school curricula - and in particular for the way school mathematics is taught.

That many people believe this conclusion is another fact no-one would dispute. But the inference - from the impact of new technology on society, to the idea that maths teaching for beginners should reflect this change - involves judgement, not logic. Exercising judgement is fine as long as one remembers that judgements can be wrong! The dangers of parochial overconfidence (reminiscent of Gulliver) are well illustrated in the following extract:

> "In view of Japan's reputation as one of the most technologically advanced nations of the world ... it came as a great surprise to find that there is relatively little calculator or computer provision in schools ... Calculators in schools are [c. 1987] almost non-existent ... According to the [Ministry], calculators are currently used from the 5th grade of Primary schools (ages 10/11) and beyond i.e. not before pupils have become competent in basic arithmetical skills using pencil and paper techniques. The potential of calculators for aiding understanding seems to have been overlooked or ignored. We saw no evidence of their use in any of the schools we visited. Our impression is that whilst calculators are widely used outside school, their use for calculation in mathematics learning is generally frowned upon." [1]

Many countries with a strong tradition of *mathematics for all* feel that fluency in arithmetic is best achieved in a relatively traditional way. Instead of shaking our heads in disbelief, we should ask *why?*. In contrast, countries that have failed the majority of their students find the idea of a 'technological fix' highly attractive.

There are still those in the UK who echo the Cockcroft Report in asserting:

> "From all the studies the weight of evidence is strong that the use of calculators has not produced any adverse effect on basic computational ability." [2]

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When it comes to assessing *the weight of evidence*, ordinary teachers have to ask whether such claims are consistent with their own day to day experience. My own experience from fifteen years of working with many thousands of British students, and of marking many tens of thousands of scripts from thousands of schools consistently contradicts such optimistic nonsense.

It is not that calculators can never be used to good effect; it is simply that the observed effects of the way they are currently being used by most pupils in the UK demand that we stop and rethink. To get us started here are four ways in which our use of calculators has undermined the way our students think about mathematics.

1. Students (and their teachers?) do not seem to realise that the **=** symbol in mathematics has a quite different meaning from the *equals button* on a calculator.

   The **=** symbol lies at the heart of mathematics. Mathematics can claim to be correct only so long as the **=** symbol is used correctly: the symbol implies that the two sides of an equation are exactly equal, and that one can explain why they are exactly equal. Thus the **=** symbol is two-way and exact.

   In contrast, the *equals button* on a calculator is one-way, and is inevitably approximate. The user of this button wants an answer, and puts aside any concern about justification or exactness. Unlike the world of the **=** symbol, the equals button behaves more like the sorcerer's abracadabra, which magically transmutes arbitrary ingredients into an 'answer'.

2. This leads on to the second anti-mathematical effect of the calculator. Mathematics is the science of 'exact **ways** of calculating', with the method being of much greater significance than the answer. In contrast, the calculator focuses attention on the output, or answer. It is the teacher's job to help students see beyond the mere answer and to understand why methods are more important. The widespread use of calculators has led us to neglect this crucial task.

3. From earliest times, the triumph of mathematics - whether pure or applied - has been rooted in an amazing stroke of good fortune. The variety of conceivable numbers is frighteningly large. For example, the set of real numbers is uncountable. In contrast, we only ever use a small finite number of reals, and no matter how hard we try, we can only ever imagine countably many (such as the endless sequence 1,2,3...). Thus, for all practical purposes, most real numbers remain unknowable. Fortunately problems involving complicated numbers can be analysed in terms of special numbers that we can comprehend (integers, primes, fractions, and some other numbers such as π, e, $\sqrt{2}$, etc.).

   Thus, for the human being trying to make sense of the universe, some numbers are more interesting than others. And the only way to achieve meaning and insight is to interpret and simplify complex events in terms of simpler things we can comprehend: the complex expression 187/374 can be mastered only by realising that it can be simplified to something familiar. Only by becoming familiar and fluent with 'special' numbers, by learning how they are related to each other, and by always simplifying the unfamiliar into familiar form can we gain control of our lives. Understanding and insight come only through simplification: this is the way of mathematics. In contrast, premature use of calculators allows students to 'succeed' without learning this fundamental lesson.

4. One result is that students become satisfied with uncomprehended 'answers' instead of insisting on meaning. Far from underlining the fact that some numbers are more interesting than others, the outputs from a calculator tend to encourage the belief that all numbers are equally uninteresting.

5. These three distortions are related to a fourth: the belief that a mathematical problem has been 'solved' once one has specified, or implemented, a sequence of steps - or button presses - which lead to the 'answer'. But mathematics is about insight, not mere algorithms or answers. Of course an algorithm can provide limited insight; it may even produce an answer. A recurrence relation may allow one to calculate the tenth, or the hundredth term. But we cannot stop there, for pupils need to learn the much greater importance of obtaining a closed form solution for the general case. Algorithms have to be used carefully. Students' algorithms are usually messy, often flawed, have no standard form, and cannot be easily checked or corrected. Much worse, there is no algebra of algorithms that might allow one to treat algorithms as mathematical entities, which can be combined, simplified, and apprehended.

References

[1] (1988) Final Report of the Mathematics Working Group, London, DES

[2] (1982) Mathematics Counts, London, HMSO

There is a response to this article, written by Kenneth Ruthven.