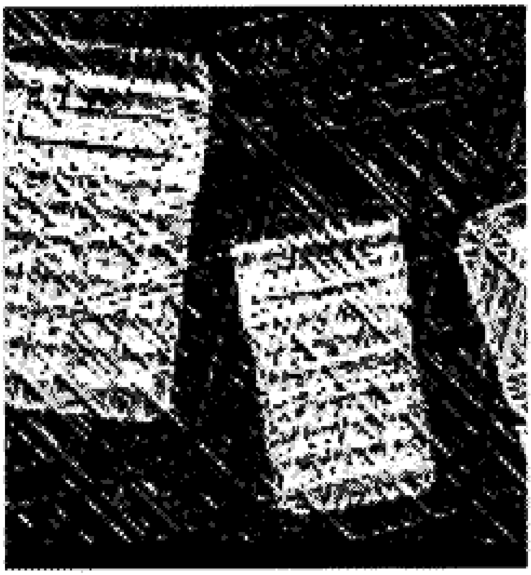
Computational tools and school mathematics

Kenneth Ruthven

The range of mathematical purposes for which computers and calculators can be used is now relatively well known, with the recent Mathematical Association report on *Computers in the Mathematics Curriculum* providing a particularly thorough survey. Here, my concern will be with a more general analysis of the ways in which computational tools can be used in the course of thinking, teaching and learning; in particular, with a distinction between their use as convenient accessories, essentially external to processes of thinking, teaching and learning themselves, and uses in which interaction with computational tools is central to these processes.



Working implement, thinking support, teaching aid

When teachers and pupils first encounter any computational tool, they largely assimilate its use to their traditional practices. For them, the tool becomes a *working implement* used to carry out already familiar processes more rapidly and reliably. Typically, for example, where previously they would have used a mental or written procedure to calculate or graph, they now use its automatic counterpart. Their use of the tool remains traditional in the sense that their broader strategies for tackling tasks remain essentially unchanged: the tool is simply a convenient means of implementing isolated processes within these strategies.

But working practices may change as teachers and pupils accommodate the use of a computational tool; often in small ways initially, but in ways on which more substantial change may build later. In effect, the role of the calculator in solving mathematical problems may shift from one in which it is simply used to execute already formulated processes, towards one in which it is used more interactively to formulate as well as execute processes, often creating novel mathematical strategies; in short, as a *thinking support*. Equally, the role of the calculator in teaching mathematics may shift from one in which it is simply used to provide information within an otherwise unchanged teaching approach, to one in which ideas are presented in novel ways in which interaction with computational tools has become a central part; in short, as a *teaching aid*. It is this transformational potential of computational tools that I hope to illustrate in the following sections.

Carrying out and checking calculations

In the classroom, the calculator is widely used to carry out calculations. It comes as no surprise that, used in this way, it is an effective and reliable working implement. Evidence to this effect comes from studies of representative samples of pupils at the end of primary school, conducted by the Assessment of Performance Unit. The availability of a calculator has a marked effect on the proportion of pupils executing a calculation correctly.

Most teachers emphasise the importance of checking calculator answers; indeed, often much more so than when pupils use less reliable mental and written methods! Nonetheless this emphasis on checking seems to have broader beneficial effects. Perhaps the most commonly taught strategy for checking a calculator answer is to compare it with a rough mental estimate. Another, rather less frequently taught, is to reverse the calculation on the calculator, working back from the answer towards an original figure. These checking practices also seem to be effective, perhaps unintentionally, as teaching aids. Again, findings by the Assessment of Performance Unit, shown in the accompanying tables, illustrate this.

1. 313 children are going on a coach trip. Each coach can carry up to 42 passengers. How many coaches are needed for the trip? Before you use the calculator to find the answer, can you tell me about how many coaches you think will be needed for the trip? Just make a guess, I only want a rough answer.

Response	*Calc used at school*	*Calc not used at school*
7 or 8	32%	18%
Other	68%	82%

2. How could you get back from (pupil answer to 21x17=) to 21?

Response	*Calc used at school*	*Calc not used at school*
Divide by 17	38%	29%
Other	62%	71%

Two items from APU practical assessment at age 11

The evidence in the table above compares patterns of response between pupils according to whether or not they used a calculator at school. The results suggest that experience of using a calculator is associated with greater success both in estimation and inversion. What we see here is how a simple refinement to the use of the calculator as a working implement seems to have developed its potential as a teaching aid. Availability of the calculator alone, however, may not help pupils to carry out calculations reliably, as the table below shows. Although around 70% of pupils seem to have carried out an appropriate division on the calculator, most have not been able to interpret the result adequately in terms of the original problem.

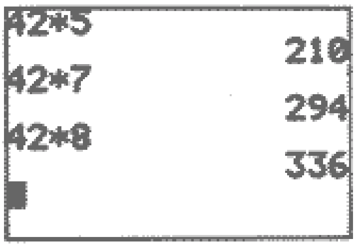
313 children are going on a coach trip. Each coach can carry up to 42 passengers. How many coaches are needed for the trip? Before you use the calculator to find the answer, can you tell me, about how many coaches you think will be needed for the trip?

Answer with calculator available, following estimate.

Response	*All respondents*
8	7%
7.4523809	49%
7	13%
0.1341853	6%
Other	25%

Item from APU practical assessment at age 11

Interestingly, despite being asked to make an estimate before calculating, there is no mention in the APU report of pupils building a calculation strategy on this estimate. Making a prior estimate can develop into the mathematical strategy of trial-and-improve in which the estimate is progressively refined as shown on the graphic calculator screen below. Such strategies have been widely observed, and are now recognised in the National Curriculum. Here we see technology acting as a thinking support, making a quite new calculation strategy possible in which interaction with the machine provides rapid feedback influencing the development of the strategy.

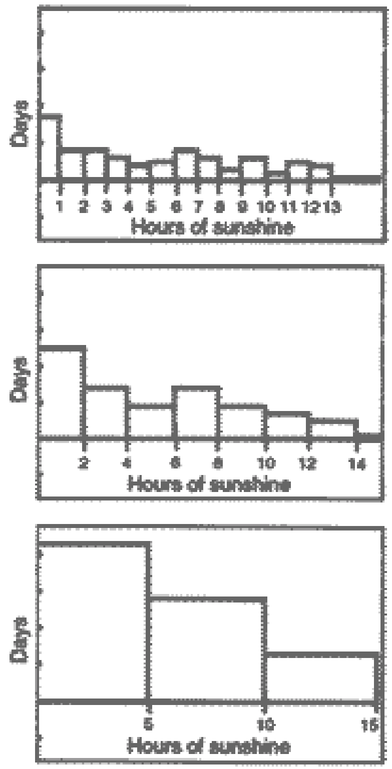


Using a graphic calculator for trial and improvement on the APU bus problem

Analysing and presenting data

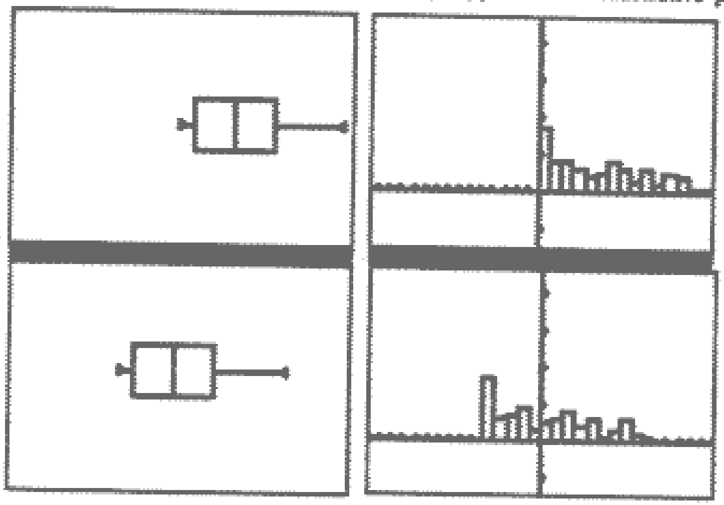
One of the most popular uses of calculators and computers is in analysing and presenting bodies of data, often gathered in the course of surveys or experiments carried out by pupils. Although there are interesting accounts of such work and its potential, and it is now a requirement under the National Curriculum, there is as yet no systematic evidence about its effects on pupils' mathematical performance. For that reason, my discussion will have to be more speculative, reflecting my own experience.

Again, a computational tool (such as a calculator, spreadsheet or database with graphing facilities) is an effective working implement, making it possible to organise, present and analyse data quickly and accurately. The ease with which the presentation of data can be modified using computational tools supports new strategies for analysing data. The graphs in the accompanying figures all relate to the same dataset: the number of hours of sunshine recorded daily at a weather station in Sheffield over a period of 84 days in summer.



Modelling the distribution of sunshine records

The figure opposite shows how pupils might work progressively towards the identification of a simple summarising model for the data. (The graphic calculator screen displays have been annotated to help interpretation.) The first graph groups the data in intervals of width 1 hour; the second graph in intervals of 2 hours. Neither establishes any simple trend in the data: the graphs are 'messy'. By exploring the effects of varying interval width, it proves possible to find a clearer summary model. The third graph indicates that on roughly half the days there were between 0 and 5 hours of sunshine; on roughly one third of days, between 5 and 10 hours; and on roughly one sixth of days, between 10 and 15 hours. This, then, is a simple form of exploratory data analysis made possible by use of the computational tool as a thinking support.



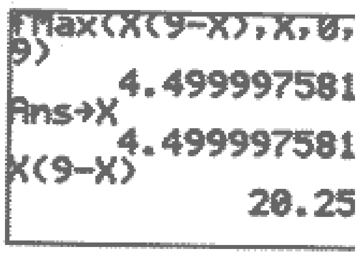
Effects of rescaling on data representation

Such use of the technology can also be developed so that it acts as a teaching aid. For example, the figure above shows what happens when the sunshine data are rescaled by subtracting the median value. At the top of the figure are the box diagram and bar chart representations of the original data; immediately below, the corresponding representations of the rescaled data. The effect on the box diagram has been simply to shift it to the left. Although the bar chart, too, has been shifted to the left, the pattern of bar heights has also changed; encouraging us to ask what exactly is going on here: essentially an effect produced by changes in the grouping of data items. This illustrates the potential of computational tools to support teaching approaches incorporating processes both of discovery and conflict.

Graphing and analysing expressions

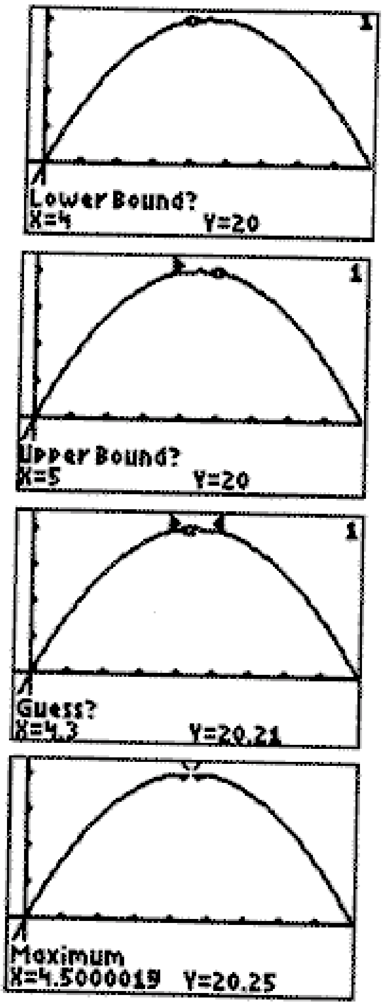
There is already strong evidence from studies at upper secondary and tertiary levels that computational tools which allow the graphing, tabulation and analysis of algebraic expressions can support innovative teaching approaches with important positive effects on pupils' thinking and learning. As yet, however, there is little research at earlier stages; consequently, my example should be treated as more speculative, although it reflects the kind of approach which has been successfully adopted in pilot studies, and is wholly consistent with the emphasis in the present National Curriculum on the numeric and graphic aspects of algebra as well as the symbolic.

For illustrative purposes, consider a simple mathematical problem: What is the largest area that a rectangle with a perimeter of 18 units can have? Taking the length of one type of side as *x* units and the other as *9-x* units gives an area expression of *x(9-x)* square units. A graphic calculator can be used as a working implement, as shown below, to find where an appropriate maximum occurs and to calculate its value.



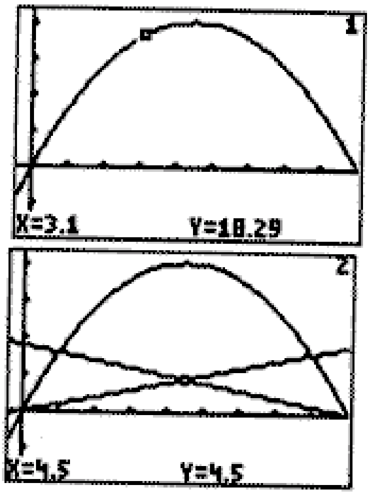
Direct maximisation of rectangle area expression

An alternative approach is shown in the figure below. Here, the expression is graphed first, giving some idea of how it varies over the appropriate range, and indicating the approximate location of the maximum. Then, by selecting a suitable menu item, the user is taken through the following steps. By manipulating a cursor on the graph screen, you are asked to choose the lower and upper bounds (in this case, selected as the ends of the segment where the maximum appears to occur). Next you are prompted to move the cursor again to give an estimate of the maximum position. Based on this estimate, the calculator makes an iterative search, and returns a value for the maximum, as shown in the final screen. Here technology is acting as a thinking support, with decision making taking place progressively in the light of cues and evidence offered by the machine.



Graphically-mediated maximisation of rectangle area expression

Equally, the graphic calculator can be used as a teaching aid in relation to this situation, as shown in the last figure. First, the process of tracing along the graph of the expression can be used to convey the way in which its value is varying, and indeed as a simple method of locating the maximum. Second, superimposing the graphs of the expressions for the two rectangle lengths makes it possible to highlight the relationship between the position of the maximum and the position at which the length expressions assume equal values. Finally, the dynamic covariation of the three expressions can be illustrated in numeric terms by stepping through a table of their values. This interplay between dynamic graphic and numeric representations of a situation is one key element of the use of computational tools as teaching aids.



X	Y1	Y2
4.2	20.16	4.8
4.3	20.21	4.7
4.4	20.24	4.6
4.5	20.25	4.5
4.6	20.24	4.4
4.7	20.21	4.3
4.8	20.16	4.2

Y1=20.25

Graphically and numerically mediated analysis of an area expression

Conclusion

These examples have been chosen from different areas of mathematics to show how the computational tools provided by calculators and computers can be used not simply as working implements, convenient accessories to implement traditional practices within school mathematics; but that they can become central components within novel approaches to thinking and teaching, in which interaction with the computational tool is the key to its use as a thinking support and a teaching aid. It is important that these more radical uses of computational tools should now be evaluated.

Kenneth Ruthven works at Cambridge University.