Shopping with Spreadsheets

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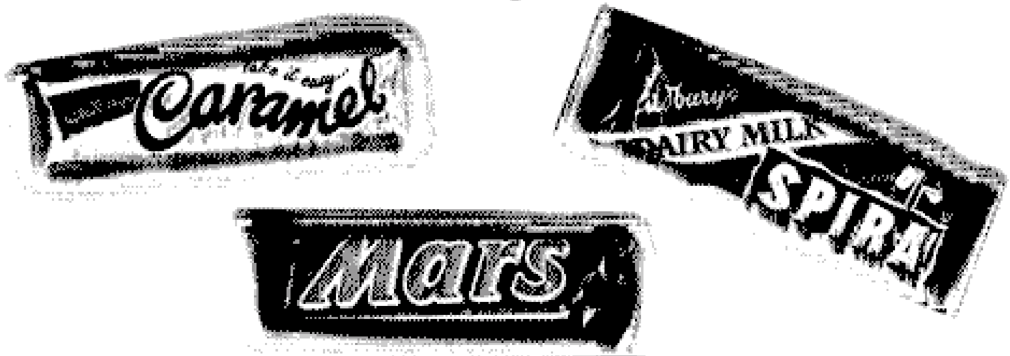


As tutors on a secondary mathematics PGCE course we work with our students on ideas of permeation of Ma1 across the other attainments targets. NCC [1] offers a description of Ma1 in three strands:

1. applications (*though we prefer making and monitoring choices*)
2. communication
3. reasoning, logic and proof

Our view of Ma1 is that:

- some aspects are possible (and should be so) within all mathematics problems;
- simplistically, the more choices that the pupils are required to make, the more opportunities there are for Ma1 to be in evidence;
- that the role of the teacher is vital to Ma1 happening in the classroom.

One of the tools that we explore with the students is the spreadsheet and its use in the teaching and learning of mathematics. We have been working with a spreadsheet to encourage mental arithmetic, spurred on by some of our students who think that, like calculators, using a spreadsheet to work on some mathematics would make pupils arithmetically indolent! Additionally we were interested to explore how the use of a spreadsheet would encourage working on aspects of Ma1.

The question was could we find tasks in the spreadsheet environment that would be successful in both these areas?

We trialled our first ideas with teachers on one of our professional development courses. One problem in particular created a wealth of comment, so much so that we were encouraged to explore some of the issues further with some school pupils (10 /11-year-olds) and some PGCE students at the university. The problem was given as a template, in the form of a shopping list with prices. For the pupils, the first problem involved buying chocolate, initially with £1 to spend, then spending up to £7 for a party.

Item	Number	Price	Subtotal
Mars	3	30	90
Twix	1	20	20
Spira	2	22	44
Caramel	5	23	115
		TOTAL	269
	Number of items		11

A second problem was given to some pupils, and to the grown ups. It was a second shopping list, as below, and those using the template were asked to spend a total of £8.

Item	Number		Price		Subtotal
Cheese	1	lb at	97	p/lb	0.97
Bread	3	loaves at	52	p/loaf	1.56
Eggs	3	boxes at	47	p/box	1.41
Beans	2	tins at	43	p/tin	0.86
				TOTAL	4.80

When we designed the task it was clear to us that it offered opportunities for such mathematics as mental arithmetic, addition, difference and estimation, this last often being a difficult area to provide practice for. Additionally it seemed to offer attributes of Ma1 such as using trial and error, justifying working, and being systematic. Also a number of choices had to be made by the solver about

- considering it as a real problem or as pure mathematics
- methods of solution
- types of calculation
- breaking down the task into more manageable aspects
- how to justify decisions to others.

By asking people to work in pairs we not only hoped to follow the working used, but also to consider what aspects of mathematical communication were part of the process of finding a solution.

With the adults, each group was asked to keep notes as they were working, on the reasons for their chosen methods of working and decisions about their search for solutions. We also made notes of the conversations that we overheard or joined in with. We observed the children working in pairs on the problem, recorded their discussions and kept notes on what numbers they chose to display on the screen. (The observer was treated as and acted as a teacher during the time the pairs worked on the problem.) We found strong parallels between the responses of the three groups. Many of the approaches were similar, as were the self-imposed restrictions. The adults were able to articulate more clearly their justifications for their choices, but the children used the context quite clearly to explain why they made their decisions. All three strands of Ma1 were evident in the processes the problem solvers used on their way towards the solutions.

The role of the teacher was vital in maintaining the possibilities within the problem. This sometimes took the form of being non-committal, in order to provoke the solvers into making their own decisions, perhaps offering approval if the solvers wished to take a risk, or, more problematically in retrospect, offering reasons to preserve the illusion of the context, especially with the children. Many questions were asked about the context of the question. The cues given in the problem imposed constraints on the solver and allowed certain choices to be made. For example, about the problem,

what kind of solution is wanted? ... are they the amounts that I would buy myself?

to which came the reply, "...you may decide."

The Y6 pupils had similar kinds of questions and comments and these provoked teacher type responses.

Pupils' comment: *Do I really have to have 6 loaves of bread?*
Observer's response: *Well, suppose it's not your shopping list.*

Pupils' comment: *I don't like eggs.*
Observer's response: *Do you like cake? ...you would need eggs if you wanted to make a cake.*

Pupils' comment: *This cheese is very cheap.*
Observer's response: *Yes, its on special offer.*

Pupils' comment: *Are you really going to spend £2 on chocolate?*
Observer's response: *I'm buying them for a children's party ...*

Other questions were asked, by both adults and children, about the nature of the solution.

Do I have to buy them all? ?

Can I have zero?

The context of the problem appeared to remain very important to most adult and child solvers. Looking for real solutions sets up beliefs about the types of choices to be made about the mathematics? For the Y6 pupils the observer/teacher colluded with them to become an interpreter of the context. Was this done to keep them on task? Could they have coped with making their own decisions about the questions they were asking or were most of their energies focused on understanding the newly learned spreadsheet environment? Or is the seeming need for a reality, within a question that appears to be real-life, essential for maintaining the momentum towards a mathematical solution?

As expected the spreadsheet provided a calculating environment that allowed the user to play with numbers. It provided access to the problem for everyone, in a way that working with pencil and paper does not appear to do.

I found that it was easier to solve the problem working on the screen rather than scribbling on bits of paper.

It was nice to take away the number crunching drudgery and concentrate on what was happening to the total.

Removing the focus from the calculations enabled the solvers to work mentally without the stress that this can sometimes cause. It was clear that all the solvers were involved in a lot of mental arithmetic, estimating, approximating, rounding, using multiplication tables and fractions.

What happens if I double all the quantities?

I found myself checking all the totals mentally.

I need another 24 pence, because we've spent that.

The main strategies for solution were trial and error or trial and improve. (On the whole the adults involved apologised for using this method: a few tried to find an alternative, 'proper' method of solution.) But other techniques emerged, such as connections between the numbers, which began to feed into the solvers' working.

Increasing beans by one and decreasing eggs by one would reduce the total bill by 4p.

Eggs and beans together make 90p.

Another method was the use of target numbers. These were numbers that could be easily achieved, and used as a basis for further trial and error.

I can get £8.10, 9 eggs and 9 beans, shall we start from there and try to get rid of 10p?

Most of the solvers used positive integer values thus restricting the possible mathematical outcomes, but even when one of the solvers strayed from integer values it was interpreted and justified within the context of the problem:

1.2 lbs of cheese – I would get this by asking for about a pound and a quarter of cheese.

Sometimes even more unlikely options appeared. One of the children when solving the problem had 0 cans of beans, but wanted to remove another 43p.

What is the number below zero? ...Oh yes, take away 1.

-1 was typed in and the solution got nearer to £8. The child was asked what this meant:

Oh, I've taken a tin back to the shop!

Why is the reality preserved? We are sure both adults and children recognise that they are doing mathematics, not going shopping, so why does the context appear to have such a strong hold on their thinking?

As all of the adults solving the problem were fairly confident with number it was only with the children that differences of performance could be seen to be related to how the children moved from the context to the numbers. Those pupils who solved the problems fairly quickly were seen to use the names of the chocolate bars and the prices interchangeably:

I'll have two Twixes that's forty pence, now give me another 30p, that's a Mars.

So another £1, that's five twenty pences, so another five there ...that's it in the Twix row.

Whereas those who only talked about the chocolate seemed to be slower at getting to the required total.

Give me five Mars, and a Twix. So we have some money left, so another Mars.

There appeared to be little difference between the children in their abilities to calculate totals or money left when asked, but the ability to move in and out of context appears to be useful when dealing with the problem efficiently. In many mathematics situations the ability to ignore the context is very important. It may be this ability to move in and out of context which we test with word problems, rather than the ability to do the calculation. But why did the observer feel it necessary to reinforce the context? Why do the solvers stay with it, even when they are breaking into more mathematical solutions?

Throughout the solving, explanation and justification were very important for all groups. In all the discussions we heard, the problem solvers explained to each other why they made certain decisions when they were confident about the step to be taken, or would offer "Let's just try it" when they were guessing. Calculations became public within the pairs. Is it simply working in pairs that enables calculations to be discussed? Or is it the nature of the display, which encourages pointing and talking? Or is it the fact that spreadsheets enable you to make predictions, but the detail of the calculation is dealt with, so that you can be brave with the trials you make?

One of the difficulties for the adults was accepting trial and improve as a legitimate strategy. In the teacher groups this gave rise to heated discussions about what they might expect from pupils in this area and what the words in Ma1 might mean. The teachers accepted the question as valid (is this the power of the teacher?) but were often uncertain as to whether they were really doing mathematics by using guesswork. What is a proper mathematics problem, what is a proper solution? Perceptions belong to the solver, or to how solvers think they are being judged. Do I, as the solver, engage with the problem and if so, which bit do I engage with? Once I have engaged with the problem are there certain behaviours that I will exhibit according to my understanding of what is expected? For example, one interpretation of this problem was as follows.

I wasn't really interested in the problem as a real problem so I altered everything but one of the items to zero. Then by trial and improvement, I found the amount of cheese to make exactly £8 (8.2474226804 lbs) (which I actually found by trial and improvement rather than by division) and then the bread (15.384615385 loaves! wonderful). Realising that the solution set was infinite, I then used negative values (but felt very subversive at the time, enjoying imagining how -3 tins of beans might be interpreted in a shopping list – must be my upbringing!) In discussion with Pat later I realised that I had been looking for solutions of the form

$$97a + 52b + 47c + 43d = 80$$
(for all $a, b, c, d \in R$*)*

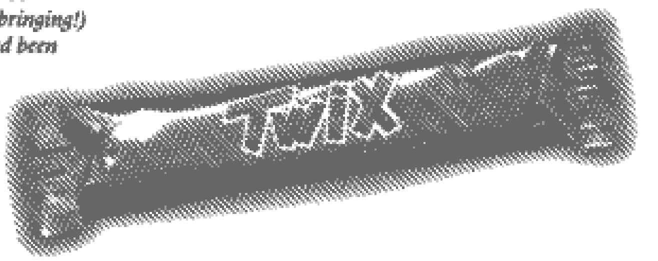
and that in fact this algebraic statement was the complete solution.

I realise now that I was confident to make my own decisions about the problem, to interpret it out of reality. I was confident to perceive it as a situation to investigate rather than a particular problem to solve though I was still working towards a solution that would satisfy the problem setter.

This approach was unusual, and yet it is the stance which one is expected to adopt with most mathematics word problems, that is, ignore the context and just deal with the numbers. In the open task the 'reality' of the context can prevent the solver from considering mathematically possible solutions.

This task has provided us with lots of new questions which we are now working on. How does context affect the development of the processes in Ma1? How might working on Ma1 help pupils to extract the mathematics from a context more easily? Teacher intervention is vital to Ma1 as in all aspects of learning mathematics, but when do we help pupils to become their own interveners, able to identify and obtain information necessary to solve the problem? Why do the adults need reassurance, even when they know they can do the mathematics? Was it unfamiliarity with the problem or the spreadsheet that undermined their confidence? Or does the difficulty lie in the nature of problems where there is more than one answer? If it is unfamiliarity with the spreadsheet, this did not deter the children. Perhaps it is our training in mathematics, which is all about getting one right answer, which prevents us from being bold with the multi-layered problem.

Back to the original question. We think that with this task we have convinced others that a spreadsheet is a helpful tool for encouraging mental calculation. We have convinced ourselves that using the spreadsheet as a tool for solving this problem offered lots of opportunities for all three strands of Ma1.



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Reference
1. NCC (1992), *Using and Applying Mathematics Books A & B*, York: National Curriculum Council.