Unpacking MaxBox

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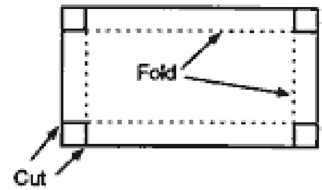
Take a piece of A4 paper, cut squares from its corners, and fold it to make an open box



Experiment with making other boxes, cutting out different sized squares. *Which box has the largest volume?*

A topic on pollution, which included looking at waste and packaging, provided a good opportunity to introduce MaxBox to some Year 7 and Year 5 children.

The children were fortunate to have high levels of access to computers, and were familiar with using a spreadsheet to record data from practical activities. We wanted to use this activity to focus their attention on using scattergraphs as an analytic tool to help them with developing experimental work, rather than just as a way of presenting results. We introduced the activity using an active graphing approach, which we have described previously in *Micromath* [1].

The children were shown the basic design for producing boxes, and everyone made a box, not worrying too much at this stage about particular dimensions. There was then some discussion about how much each box would hold, during which it became clear that most of the children had had some experience with using the formula length x width x height to calculate volumes. As understanding volume wasn't the main objective of the activity, we were happy to gloss over this, and simply encourage the use of the formula.

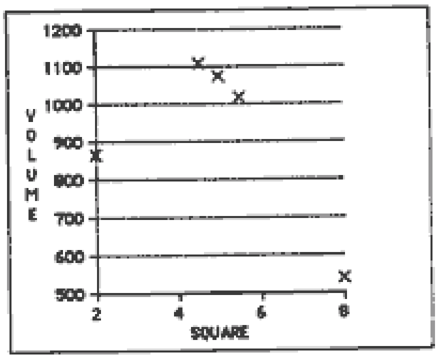
Several opinions were expressed about the relative volumes. Quite a number of children felt that since they had all used the same size paper to start with, the boxes would all have the same volume. One group stacked some of the finished boxes inside each other, and suggested that as the shallowest box could 'hold' all the others, it must be the biggest. Others supported the merits of deeper boxes, and quite a few were unwilling to express an opinion!

Once everyone had agreed on which measurements were needed, and how to calculate the volume, we introduced the layout for recording their results on a spreadsheet.

	A	B	C	D	E
1	Size of square	Volume	length	width	height
2					

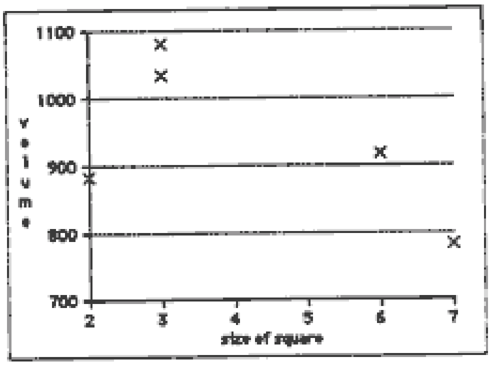
Normally, we are not so prescriptive and leave children to design their own layouts, but on this occasion we wanted them to be able to graph the size of the square cut out against the volume, and that meant that they needed to have this data in adjacent columns. Several children were confident that they could enter a formula for the volume, and fill down the column, and they were given the responsibility of helping anyone who got stuck.

The children then got into groups, measured their own boxes and entered the data. Once they had four or five boxes recorded, they were asked to make a scatter graph, and then discuss what to do next [2].



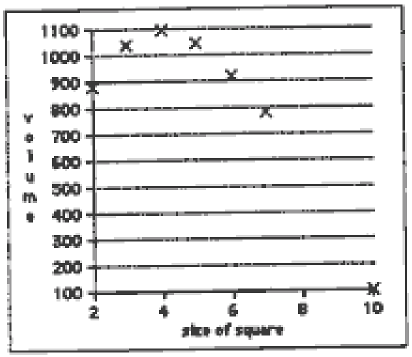
Listening to these discussions, it became clear that opinions about the relative volumes were changing. Everyone now agreed that the volumes were not all the same! The graphs produced by some of the groups suggested that cutting out larger squares would increase the volume; others were less clear about the pattern they could see. A few groups were beginning to suspect that middle-sized squares gave the biggest volumes. There was a lot of discussion about what to do next.

Jenny's group



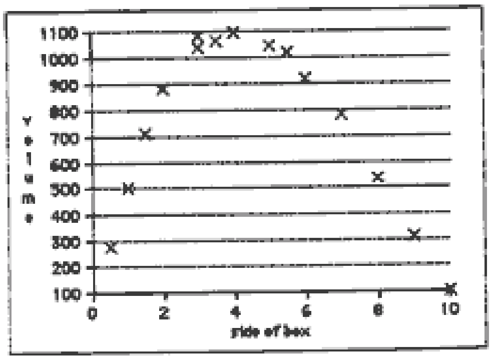
Jenny's first graph

Some of the graphs, like Jenny's shown above, showed some anomalies: after some discussion, the group decided that the two crosses vertically above each other both showed the data from boxes made by cutting out a 3 cm square. They realised that they had two different sets of measurements for boxes that should have been identical. They went back to the spreadsheet, checked their measurements and corrected their data. Then they decided to make more boxes, using squares of between 3cm and 6cm to fill up the gap in their graph, and also to try some larger squares. The girls in this group enjoyed making the boxes, and worked very carefully and accurately. Their next graph showed the pattern of the results more clearly.



Jenny's second graph

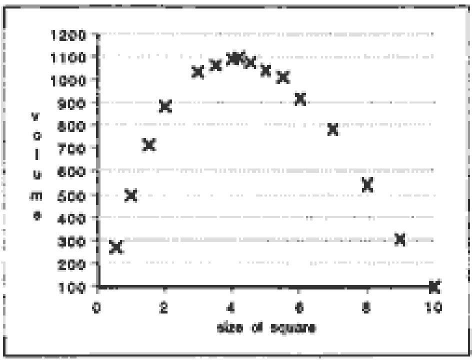
They were impressed with the shape of the graph, and decided to collect more results to 'fill in' the graph. The accuracy with which they made and measured their boxes is impressive!



Jenny's third graph

Finally, we encouraged the girls to go back to the original problem, and try to find the size of square which gave the maximum volume. They realised that the highest point of the graph would lie between 4 cm and 5 cm, and they began homing

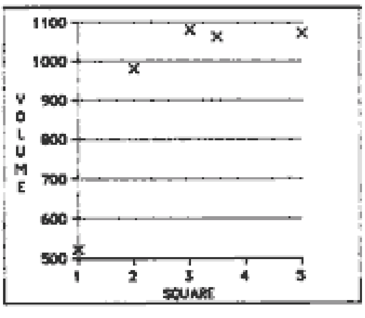
When we had done a few boxes we made a graph out of them, the graph was a scatter graph.
We found that we had many gaps in their so we decided to fill them up.
There was already a definite pattern in where the x's were marked.
We put in a few more x's to fill up the gaps, the scatter graph looks now like a hill. Look below.
We found that the bigger and smaller sized squares did not have a big volume, but the middle sized ones were the tallest. Our biggest volume of a box was one with a 4 cm square.



in, cutting out squares of 4.5 cm and then 4.2 cm. Their final report sums up their findings(see above).

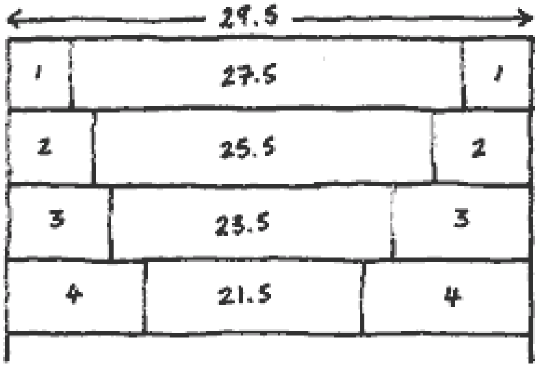
Ruth's group

Not all groups found it as easy to produce accurate results by making and measuring boxes as Jenny's group did. Ruth's group collected data from five boxes before they drew their first graph. This did not show a clear pattern and the group had enough experience of reading graphs to realise that their data was probably not very accurate.



Ruth's first graph

They wanted to produce a smoother curve, and finally decided that perhaps they did not need to actually make more boxes. They measured the length of the A4 paper to be 29.5 cm, and drew a diagram to help them work out what the length of the box would be for various sized squares.



They used a similar diagram for finding the widths, and were struck by the patterns they could see in the way the length and width decreased. They entered some of this data into the spreadsheet, and were pleased with the improvement in their graph. However, we wanted to push them a little further, and suggested that they should get the spreadsheet to do the calculations for them. Their method was;

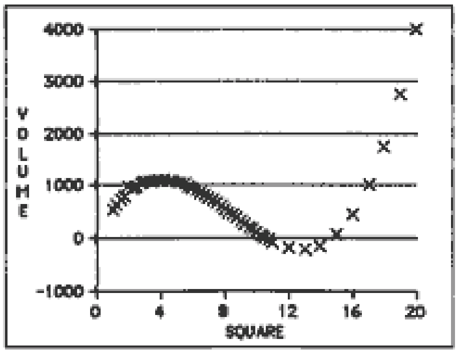
start with 29.5, (the length of the paper) and take away twice the size of the square.

With some support, they were able to translate this into formulae on the spreadsheet.

	A	B	C	D	E
1	Size of square	Volume	length	width	height
2					
3					
4		= C4*D4*E4	= 29.5-2*A4	= 21-2*A4	
5					

Interestingly, they got really stuck when we suggested that they put a formula for the height of the box so that they would only have to type in the size of the square to get all the data. They clearly had the idea that a formula had to do something to the number: just copying it felt wrong.

Once they had their formulae in place, they filled down the columns of the spreadsheet, and typed in a range of sizes for the square. They tried various small increments to try to produce a really smooth curve on the graph, and put in some larger values as well. They were surprised and impressed with their final graph.



Ruth's final graph

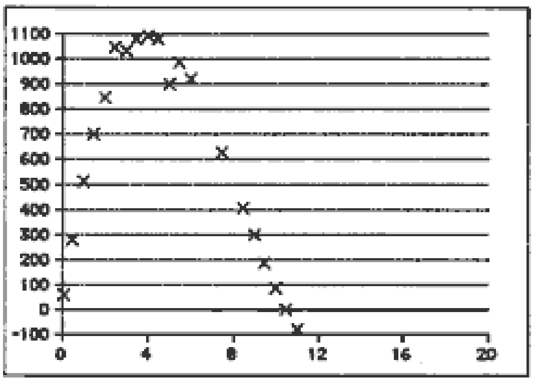
Michael's group

Other groups also realised that they could harness the power of the spreadsheet to tackle the problem without having to make the boxes. Michael's group moved quite quickly to using formulae: they knew quite clearly what they wanted to do, and only needed a little help from us to translate this into a form that the spreadsheet could understand. The real stumbling block for them was that, unlike the previous group, they expressed their method by starting with the size of the square:

double the size of the square, and then take that away from 29.5

The problem was that they couldn't type the instruction take that away from 29.5 into the spreadsheet, and it took some time for them to re-formulate their method.

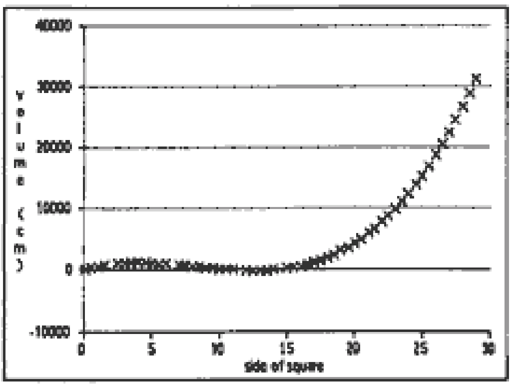
Once they had filled the formulae down the columns they started entering different numbers for the size of the square, and were intrigued when some of these gave zero or even negative values for the volume of the box.



Michael's graph from a mixture of measured and calculated data

Liam was able to explain the zero value for the volume by saying that for that box they would have cut away two squares that met in the middle, so there would be no width left. Denise said that they would just be folding the paper in half, so there was no volume at all. The negative values were harder to explain, but the group realised that these were impossible boxes, since the two squares that had to be cut away were larger than the whole width of the paper.

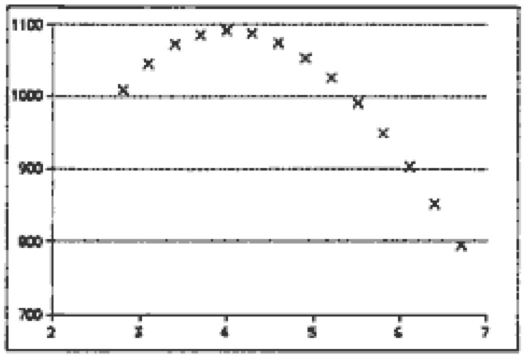
The shape of this graph encouraged them to go on and use the spreadsheet to produce more data, and see what happened. By now they were happy to work with the computer model they had created, and weren't thinking too much about the boxes. To save time, they used a formula to increase the size of the square by 1 cm each time, and filled down more and more rows. The graph they produced was another surprise.



Michael's final graph

The immediate reaction was 'Where has the hill gone?' As they explored and discussed this graph they began to get a sense of how scale had affected its appearance, and they managed to pull it around until the hill re-appeared. We talked to the group about what they thought was happening, and what might happen as the graph continued. They had some ideas about the impossible boxes becoming more peculiar as the overlap between the squares that got too big to cut out increased, but it is hard to recapture them in coherent sentences, without the eloquent hand movements that accompanied the discussion. They thought that the graph might have another curve in it later on, or get to a certain point and simply not get any higher, but no one suggested that it might just go on increasing. In fact, they made several graphs on the way to this one, extending the columns of the spreadsheet to include more data each time, but as they did this the effect of the scales meant that the appearance of the graph didn't change much, so the notion that the graph might simply stop and not go any higher was an understandable one.

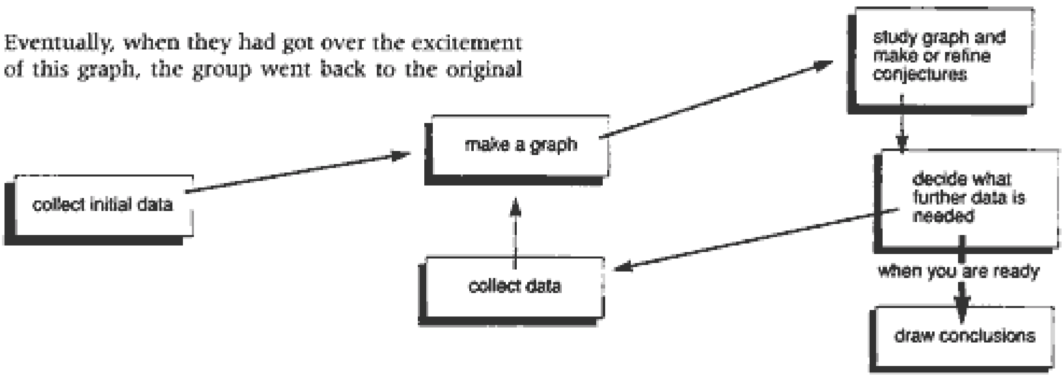
Eventually, when they had got over the excitement of this graph, the group went back to the original problem, and used their formulae, with a smaller increment in the size of the square, to home in on the optimum value for the volume.



Michael's graph homing in on the best value

What did we find inside?

The three examples described here are typical of the range of responses across the two age groups: Jenny's group were average ability Year 7's, Ruth's group bright Year 5's, and Michael's group some of the most able in Year 7. One of the messages for us from this experience was that there was little difference between Year 5 and Year 7 in the ways in which children tackled the activity. In general the Year 5 children were more interested in actually producing finished boxes, but the numbers of children in each class who moved from a practical to a theoretical approach were roughly equal. A significant advantage of the approach we used was that all the groups were able to make progress with the problem. Rather than using graphs only at the end of the activity to present the results of the data they had collected, we encouraged the children to use graphs in an *active* way, which we might summarise as follows.



Studying the graphs helped them to decide what to do next; it gave a structure to their investigations, and also showed up places where they needed to check and correct results. This helped to keep up the momentum of the activity without constant teacher intervention.

Few if any of the children guessed what the box with the maximum volume would look like, so there was a real purpose in reading and interpreting the graphs. Most of their previous experience had been with activities from which relationships could be shown by a straight line on the graph, and we were impressed by their ability to transfer this experience to interpret the meaning of a curve.

We had hoped that some groups would get on to using an algebraic formula to generate data, and we looked for opportunities to introduce this possibility. These seemed to arise naturally in a number of ways. For some groups, such as Michael's, the incentive was being able to generate a lot of data quickly, without the time needed to make boxes. Ruth's group was typical of a number who first wanted a way of working out results in order to check on the accuracy of their measurements. This often arose when the graph showed one point that was obviously out of the general pattern, or two different results from the same size square. Another group looked for a way to calculate the dimensions of boxes instead of making them when they decided they needed to add data about extreme cases. It proved to be difficult to make a box accurately by cutting out a 1 cm or half cm square, but by now they could work out what the length and width would be for particular cases. With some support, and encouraged by what they had seen other groups producing, they were able to translate their informal method into formulae on the spreadsheet.

We felt that the children's familiarity with the language of the spreadsheet supported this move to find, and then generalise, a method for calculating the dimensions of the box. We had not anticipated the difficulty many of them found in reconciling their way of expressing such a method (e.g. double the size of the square and take that from 29.5) and the formal conventions of arithmetic notation. However, we were impressed by the way in which most children were prepared to persevere in struggling to get this right. Tentatively, we attribute this to their ownership of the problem, and to the feedback they were receiving from the computer. These are areas we shall be exploring in greater depth in the future.

Many of the children focused on the appearance of their graph, and really seemed to appreciate the smooth curves that could be produced from calculated data. They were beginning to get a sense of the difference between measured data, which would always give approximate results, and an untidy graph, and the precision of the data generated using formulae. Michael's group were happy with their graph which was drawn from a combination of these two sources, and could point out which crosses represented real boxes which they had measured. Other groups rejected the measured data altogether. Once they had formulae which worked, they used them to replace all their previous results.

Perhaps the most unexpected outcome for us was the children's fascination with extending their graphs to beyond the range of boxes they could actually make. This was a real insight into the power of a mathematical model, but also the difficulties of making sense of the results it may produce. We end with an extract from our field notes which gives some indication of how the children tried to come to terms with this. Ruth and Anthony are using formulae for the length and width of the box, taking the length and width of the paper as 29.5 cm and 21 cm.

... they have been putting in more values, and Ruth came to find me to tell me that they had put in 10.5 and got the volume 0, and then put in 11 and got a negative volume. I asked them why this might have happened. Ruth said, 'if you cut out squares of 10.5 it would just be like folding the paper in half.' I sketched with my finger on the paper ... and they could see that there would be no width because the two squares met in the middle. Ruth then took over and drew what would happen for 11. 'You couldn't really cut it out. It would overlap. There wouldn't be enough left to cut the second square.' I suggested that there wasn't just no width, but less than no width – a negative width, so the volume was negative, and they were happy with this. They then drew a graph, and Anthony said 'Wow what a curve!'

Janet Ainley and Dave Pratt work at Warwick University.

Our thanks go to children and teachers taking part in the Primary Laptop Project at Brookhurst School in Leamington Spa.

References

1. J. Ainley & D. Pratt (1994) Runaway cars, *Micromath* 10.2, Summer 1994
2. The software we were using (ClarisWorks) produces graphs with only horizontal grid lines showing as its default, though this can be changed. However, the children never expressed any difficulties about working with graphs drawn in this way.