Geometrical constructions are time-consuming to execute. Consequently hypothesis testing is very slow. **Heinz Schumann** has been thinking about the ways in which computers might help.

The computer as a tool for geometric constructions

The geometry curriculum for pupils aged between 10 and 16 is mainly based on three interdependent subject divisions:

- geometrical constructions;
- study of geometrical forms;
- calculations concerning geometrical figures.

Traditional instruments for constructing pictorial representations of geometric figures currently used in school include:

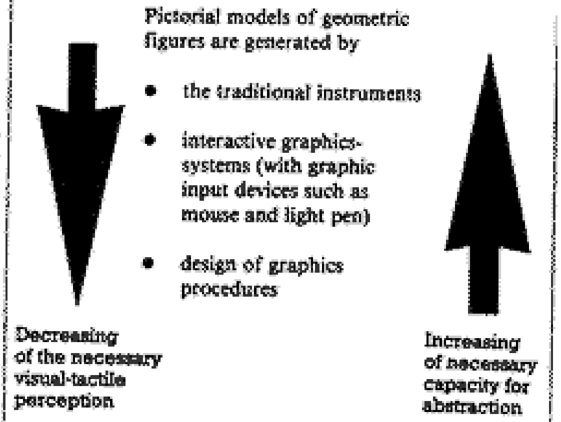
- (unmarked) ruler;
- compass;
- ruler for measuring distances;
- protractor;
- set-square.

In addition to these traditional instruments there is now the possibility of generating pictorial representations of geometric figures on a computer's screen. The following methods of generation are possible.

- The representation can be generated directly, using the commands of a graphics system, step by step alternating between command input by the user and command execution by the system (interactive constructing). The result is an electronic model of the target construction as a state of the graphics screen.
- The representation can be generated indirectly by designing a procedure in a problem-oriented language. The input variables are the given data of the construction, the result is an artificial language model of the geometric construction process; the procedure call generates the pictorial construction process on the screen.

Certain interactive graphics systems are able to make an abstraction from a special construction. This means the system can produce automatically a similar construction with changed data.

The situation is illustrated in the diagram below.



Disadvantages and advantages of the use of the computer

The following are possible objections to the use of a computer for geometric constructions.

- There might be a loss of routine skills in using the traditional instruments. (Will such skills be required in future? Does the curriculum have to be changed?)
- There might be deficiencies concerning the understanding of geometric concepts and statements. (Seen from a cognitive psychological point of view, is there a basic difference between learning geometry by computer and learning it by traditional means?)
- Is there a shift in emphasis from construction problems to interaction problems? (Does the pupil learn more about a graphics system than about geometric construction?)
- Does the use of the formalized language-oriented system hinder immediate access to evident geometric facts?
- Is there decreased emphasis on proving geometric statements?
- If use of non-standard and non-standardizable graphics ▶

systems for geometric constructions becomes common (in place of the universally understandable traditional instruments) is there a danger that the geometry internalized will be system-dependent?

- There is a danger that only those geometric objectives are preferred, which could be adequately performed by the used system (objective-selectionism).

Irritations in the visual perception of geometric figures which can arise from a deficient graphics solution, or by a lack of adjustment of the screen or of the printer, can be regarded as temporary problems.

The following are arguments in favour of using graphics systems or graphic-competent programming languages for the production of geometric constructions:

- dynamic visualization;
- the facility to reproduce as required;
- the facility to correct and modify as required;
- the quick availability of complex constructions;
- the facility to compose constructions;
- a large range of variations.

The use of graphics systems encourages the following ways of working:

- algorithmic thinking;
- modular working;
- abstract thinking;
- use of experimental or systematic procedures, depending on the type of graphics system;
- the gaining of inductive recognition.

Geometric demands

The process of construction by a graphics system must correspond, step by step, with the process of construction using traditional instruments.

The user should be able to perform the following actions (traditionally accomplished using straight-edge and compass):

- drawing a straight line through two given points;
- drawing a circle round a given centre and through a peripheral point;
- fixing the intersection points of constructed loci (lines and circles);
- selecting points anywhere in the plane, especially on constructed loci.

The user should be able to produce:

- line segments
- half lines (rays)
- arcs.

The user should be able to get the system to perform the following actions (traditionally accomplished using protractor and marked ruler):

- measuring angles;
- measuring lines.

In order to simplify the production of a complex constructions and to save time a graphics system should perform the following basic constructions via a single command:

- line through two points;
- line segment between two points;
- half line (ray) through a starting point and a further point;
- circle around a centre and through a peripheral point (or of a given radius);
- the spacing off by measurement of a line segment on a given half line;
- the drawing of an angle of given measure on a given half line;
- the mid-point of a line segment;
- perpendicular bisector;
- parallel through a given point to a line;
- perpendicular from a given point to a given line;
- angle bisector.

A graphics system should have a denoting function for geometric objects with the following capacities:

- a flexible positioning on the screen;
- use of the usual geometric terms;
- the possibility of omitting denoting.

(Automatic positioning of denoting can lead to overlapping or erasure.)

Furthermore the construction-system should aid the student by interactive

- continuously varying construction issues;
- defining of macro-constructions;
- repetition of a construction;
- generating of loci point by point and so on.

Comparative survey and analysis

There are basically two different methods for the operation of graphics systems:

- the menu-driven graphics systems, where the user selects menu options and is then questioned for further specifications (table 1);
- the command-driven graphics systems, where the user types in command words and their specifications via the keyboard (table 2).

With all the graphics systems in question their interactivity is restricted, because all basic geometric objects must be named before construction and one can only refer to these objects by using their names.

Menu-driven graphics system for geometric drawing (table 1).

Name	The Geometric Supposer (Points and Lines, Triangles, Quadrilaterals, Circles)	The Geometric Constructor	GEO	GEOPAS	TASTEN-IGEL (KEY-TURTLE)
Authors	J.L. Schwartz u. M. Yerushalmy Newton EDC, USA 1985	C. Kimberling, University of Evansville USA 1985/86	W. Settele, Hildegardis-Gymnasium, Kempten, W.-Germany 1986	B.v. Pape, Gymnasium Eversten Oldenburg/Old, W.-Germany 1987	H. Schumann PH Weingarten W.-Germany 1985/86
Computers/ Operating systems	Apple II +,e,c (64K) MS-DOS	MS-DOS	C 64/C 128 Portation to MS-DOS is planned	MS-DOS	C 64/C 128 Apple II +,e,c MS-DOS
Underlying programming language	BASIC/TRUE-BASIC	CB-BASIC	Basic/Assembler	TURBO-PASCAL	MIT-bzw. LCSI-LOGO (English/German)
Target group	American high-school pupils, class 6-12 (age group : 12/13 - 17/18)	American high-school pupils without reference to class	Pupils of German Realschule and Gymnasium, class 7- 9	Pupils of German Gymnasium, class 7-10	Pupils of German Realschule/ Hauptschule, class 7-9
Conception of use (according to the authors)	As a teaching tool a) Demonstration: introduction or stimulation of new ideas. Investigations of data, forming suppositions b) Individual or group work in computer lap: Using work-sheets-guided inductive discovery of geometric statements. Pupils write reports on their work and produce results.	Instrument to produce all (?) Euclidean constructions quickly and in the same sequence, in order to find new geometric statements (competitive discovery). Construction process and construction result helps the teacher to stimulate classroom discussion.	As a geometry-tool in the teaching of geometry. The teaching of an exact geometric way of thinking and speaking. Improvement of motivation, quicker production of complex constructions. Emphasis of partial constructions as basic constructions are available.	As a tool for geometric construction : in class 8 - repetition of triangle construction, in class 9-10 - geometric transformations and patterns. Acquisition of geometric terminology and standardization. Also a tool for the teacher.	As a primitive tool for geometric drawing. For the preparation of designing procedures for generally valid constructions.
Referable geometric objects	only points	only points	points line segments lines half-lines (rays) angles circles	only points	only points
Basic constructions as menue options	point intersection point(s) connecting line segment connecting line (implicit) circle parallel perpendicular angle bisector perpendicular bisector measure of segments and angles random points (Program Triangle: Circles, inscribed and circumscribed median altitude midsegment measure of perimeter and area)	point intersection point(s) connecting line segment connecting line (implicit) circle midpoint of a segment angle bisector perpendicular measure of line segments (additional options concerning triangles)	point intersection point(s) line segment half-line (ray) line angle circle polygon laying off a line segment protracting an angle parallel perpendicular bisector perpendicular angle bisector measure of segments and angles selection of points on geometric objects	point intersection point(s) connecting line segment connecting line (implicit) angle circle arc parallel perpendicular bisector angle bisector perpendicular measure of segments and angles	point intersection point(s) connecting line segment connecting line (implicit) circle parallel foot of a perpendicular foot of a "slant perpendicular" measure of segments and angles
Special features	Arbitrary selection of basic figures. Figure-scaling. Automatic transfer of constructions. Calculation of terms dependent on variable point position (only apple version). Automatic positioning of objects and of point denoting; alphabetical point lettering.	Diverse possibilities of constructing ellipses, parables, hyperboles. Automatic production of a construction description. Automatic interaction agenda. Automatic positioning of point denoting; alphabetical point-numbering.	Transformations of polygons. Points selection on referable objects. Automatic production of a construction description. Automatic positioning of point denoting.	Diverse transformations of polygons. Automatic production of a construction description. Options for automatic positioning of point denoting.	Transparent software. Guiding drawing pencil. Open to the programming system LOGO. Interaction agenda is optional. Automatic positioning of point denoting

Final points

- Does a graphics system for geometric drawing, suitable for use by pupils, exist?
- Does the existing school geometry curriculum offer possibilities for using computers?
- Can the aims of learning and teaching geometry be better realized by the application of the computer as a tool for geometric constructions rather than by the use of the traditional means?

At the present stage of development it must be said that the answer to all these questions is still negative. There are some hopeful prototype systems under development, which are being made compatible to school-geometry and to meet general software-ergonomic standards for tools (for example *Cabri-Geometer*). It is the author's intention to

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Command-driven graphics system for geometric drawing (table 2).

Name	KoBesch (Konstruktions-beschreibung)	TRICON (component : Lektor)	CONGEO (EUKLID)	LOGO with graphics extension	L.E.G.O. (Lisp-based Euclidean Geometry Operations)
Authors	H. KOHS/Project-team CONMAT, Mainz W.-Germany 1986	C. Holland University of Gießen, W.-Germany 1987	V. Sadolin Danish Teacher Training College Denmark 1988	H. Schumann, PH Weingarten W.-Germany 1985/86 Allard, J.-C./Pascal, C. IREM de Grenoble France 1986 G. Taske, Gymnasium Schloß-Neuhaus Paderborn W.-Germany 1987	M. Fuller P. Fraczkiewicz University of Regina, Canada 1985
Computers/ Operating system	MS-DOS Apple II +,e,c	MS-DOS	MS-DOS	Apple II +,e,c MS-DOS	VAX 730/UNIX SUN Workstation (Portation to MS-DOS is planned)
Underlying programming language	Turbo Pascal	Turbo Prolog	COMAL	MIT-resp. LCSI-LOGO with window command	Franz Lisp
Target group	Pupils of German Gymnasium, class 8	Pupils of German Realschule and Gymnasium, class 8	Pupils of Danish schools from class 6 onward	Pupils of German Realschule/Gymnasium, class 7-9; Pupils of French lycée	Pupils/students without further information
Conception of use (according to the authors)	As a teaching medium for the graphical transforming of construction descriptions especially of triangular constructions. Corrections of gaps and mistakes in descriptions. Comparison of different solutions. Experimental search for a solution strategy. Geometric statements, discovery and inductive verification.	Tool of construction for the production of special triangular constructions. Examples of exercises for learning the stages of construction which are to be carried out in the dialogue mode (the essential tutorial system of TRICON).	Tool for geometric construction and for designing graphics procedures. Modular work.	Tool for geometric constructions in the command mode and for designing graphics procedures in the edit mode. Modular work.	The computer as a wall display unit. Visualization of geometric objects and constructions. The computer as a virtual lab: the pupils/students generate and manipulate geometric objects, vary features of constructions and acquire empirical material for the setting of hypotheses.
Referable geometric objects	points line segments lines half-lines circles angles	points lines half-lines circles	points lines circles	points line segments lines half-lines circles	points line segments circles planes
Basic constructions as commands	point intersection point(s) connecting line segment line half-line circle angle laying off a line segment protracting an angle midpoint of a segment angle bisector parallel perpendicular measure of segments and angles	point intersection point(s) connecting line segment connecting line half-line circle laying off a line segment protracting an angle parallel perpendicular measure of segments and angles	point intersection point(s) connecting line segment connecting line half-line circle protracting an angle measure of segment and angle measure of shortest distance point-line	point intersection point(s) connecting line segment line half-line circles laying off a line segment protracting an angle midpoint of a line segment parallel perpendicular measure of segments and angles measure of shortest distance point-line	point intersection point(s) connecting line segment circles midpoint of a line segment measure of segment and angles (only 3-dimensional commands respected)
Special features	Automatic production of animation graphics by making a piece of data variable. Automatic positioning and scaling of objects and positioning of denoting. No facilities for the designing of procedures.	Automatic positioning of objects and denotings. No designing of procedures possible.	A procedural programming language for graphics extension is available and compatible with the command language. Definition of modular procedures.	Transparent software because of mother-language instructions and commands. Open and compatible to LOGO. Definition of modular and recursive procedures.	Possibility to make all data of a construction variable during the interactive constructing by the definition of a Lisp-function. The generation and manipulation of 3-dimensional objects. Open to and compatible to Lisp. Definition of modular and recursive procedures.

report on this development in one of the future issues of *Micromath*. ■

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