The computers used in school classrooms are digital computers. **Pat Deery** has had experience of analog computers and this led him to write a computer program to simulate the output of an analog computer. Here he describes some of the uses to which his program has been put with teachers and with students.

ANALOG

The purpose of ANALOG, like that of any enrichment material, is to unify mathematical concepts and make mathematics more meaningful to a particular individual. The best measure of the success of enrichment material would be to observe learners and answer these questions.

- Are learners involved and interested?
- Are they discussing mathematical ideas in concrete terms (or communicating ideas)?
- Are they meeting known concepts from a different point of view?
- Is "mathematics happening"?
- Would the teacher be happy to use the material again?

This article describes the use of ANALOG with: a group of A-level mathematics teachers; a class studying for London GCE A-level Mathematics; a class who had already done well in A-level Mathematics who were studying for Further Mathematics (A-level Mode 2 syllabus); and a class studying Additional Mathematics.

A workshop for mathematics teachers

When a group of mathematics teachers used the program two worked individually at their own machine, while the rest preferred to work in pairs. All quickly became involved with the mathematics. I was available to give help.

> The program ANALOG simulates the action of an analog computer by allowing the user to solve differential equations and plot the resulting output on a vertical or horizontal axis. If a "time base" is plotted horizontally then a graph of the solution of the differential equation (or its derivative) may be displayed. If the equation of the Y-axis models some physical situation the horizontal values may be kept "fixed" and the user may observe the model in action. The solution of one differential equation may be plotted vertically and that of a different equation plotted horizontally giving access to more advanced modelling situations.
>
> Extract from the ANALOG manual.

One teacher was quite happy to investigate SHM by varying parameters and seemed pleased with the graphs produced. But when changing $y'(0)$ produced a change in amplitude he expressed surprise and thought he had discovered a bug in the program. It was only after discussing the physics of the situation, and after further experimentation, that he realised that the change in amplitude was to be expected. He then experimented to see the effects produced by changing the value of c in the equation $y'' + cy = 0$ ($y(0) = p$, $y'(0) = q$) and seemed quite pleased with the results. His comment at the end of the session was illuminating.

"That was interesting. I suppose I should have known all that about SHM but I never really thought about it like that before".

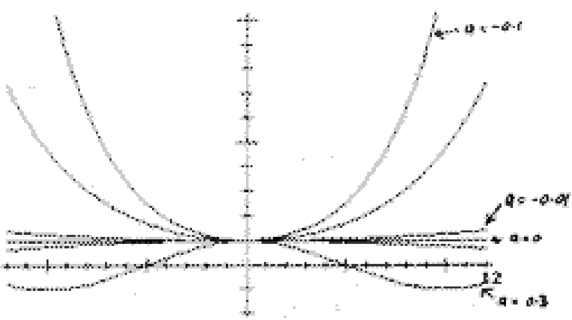


Figure 1

Another teacher suggested that a weight on an elastic spring would have provided the same insights but the first teacher disagreed and believed that he had seen SHM equations from a new point of view and would like to introduce the program to his pupils.

Another teacher mentioned that he was introducing hyperbolic functions at present to an A-level group. After much discussion it was decided to investigate the solution of $y'' + ay = 0$, $y(0) = 1$, $y'(0) = 0$, for positive, negative and zero values of a. This produced an interesting series of graphs (See figure 1). The teacher enjoyed doing this simple investigation and came to view hyperbolic functions in a new light. He decided he would like to compare the effects of drawing $y = \cos t$, $x = \sin t$ with drawing $y = \cosh t$, $x = \sinh t$, and so started on his own self-motivated investigation.

A different pair found it interesting to consider the problem of setting up a differential equation to model the behaviour of a parachutist jumping from an aeroplane, and solving it on the computer. They had some difficulty about the sign of the air resistance term in the equation $y'' = -9.8 + ky'$; $y(0) = 1000$ $y'(0) = 0$ and spent some time discussing it. Some difficulty also arose about a suitable length of time to calculate over but eventually they were satisfied. They graphed y' as well to observe terminal velocity.

Before leaving, each group demonstrated their work to the others. Most were prepared to use ANALOG in their own classroom. The fact that they had been using ANALOG to solve problems and had been actually 'doing mathematics' rather than watching a demonstration seemed to convince them of the worth of the package. Before teachers can realise the value of computers in their classrooms they must first become absorbed in using a computer to solve problems for themselves.

An A-level mathematics class

The teacher in a local school was willing to allow me two forty-minute periods to use ANALOG with her A-level maths group. This group, consisting of six good mathematicians, had no experience of computers, although one had a microcomputer at home. They seemed to regard the single BBC Master set up at the back of the room for the occasion with the same suspicion with which they regarded their new mathematics teacher. It seemed that the lesson was going to be mostly a demonstration — but nevertheless students were encouraged to type in numbers and use the arrow keys.

Once students were familiar with the working of the menu system they looked for a physical example to solve. The question about the water flow into and out of a tank from a London University A-Level Paper seemed appropriate because they had solved the problem recently.

"A water tank has the shape of an open rectangular box of length 1 m, width 0.5 m and height 0.5 m. Water may be drained from the tank through a tap at the bottom of the [illegible] is closed. Show that, t minutes after the tap has been opened, the variable h satisfies the differential equation

$$10 h' = 1 - 2h$$

On a particular occasion the tap was opened when h = 0.25 and closed when h = 0.375. Show that the tap was opened for 5 ln 2 minutes."

The equation was easily entered onto the machine and, after some experimentation, the students decided to compute for 100 minutes so that they could see the steady state solution more clearly. The fact that the height of the tank was 0.5 m and the steady state solution was at h = 0.5 drew some comment. Without any prompting one of the students asked what would happen if the tank was initially empty — and the new graph showed their predictions to be correct. The students tried some more initial heights (including h = 0.5) but no one suggested any height greater than 0.5 — perhaps because of the limitation built into the question. When this limitation was removed they seemed generally surprised and pleased that h = 0.5 was still the steady state solution, until one of them pointed out how to anticipate this by taking $h' = 0$. The ensuing discussion about whether $h' = 0$ gave a turning point, what type of turning point, and where it occurred on the graph, was concrete mathematical discussion that would probably not have developed had the students not been handling the situation in what felt like a real way.

It appeared to me that mathematics was beginning to happen when the girls were genuinely interested in the solution obtained from different initial depths of water and when the discussion of "t tends to infinity" was in concrete terms. But to derive real benefit from using any piece of apparatus or aid, it must become familiar through regular use.

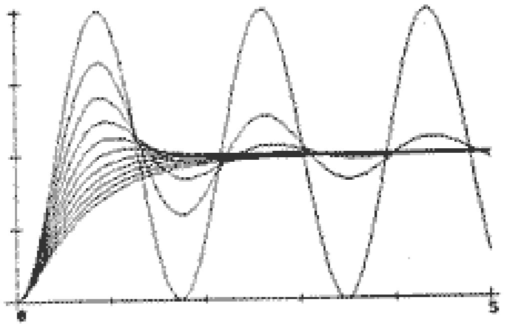
A Further Mathematics class

One of the teachers at the workshop expressed more interest than the others in using ANALOG. He had a class of eighteen studying Further Maths at A-level.

The teacher had been using an investigative approach in the classroom and was involved in the development of the Northern Ireland Mode 2 examination in Further Mathematics. He was interested in enrichment material and suggestions for student projects. The students were familiar with the use of a BBC microcomputer. They had access to a microcomputer laboratory with eight BBC Masters and it was relatively easy to bring a Master to the classroom for demonstrations or other uses.

The students had used *Function Graph Plotter* (FGP), had done some modelling using a spreadsheet and had used a statistics package. The teacher was interested in making ANALOG one of the pieces of software that students had access to for ordinary everyday use.

The teacher introduced ANALOG slowly, using it in various modes. He demonstrated it to the whole class, using it to solve differential equations in connection with SHM



Also, whenever a differential equation was being solved the solution was demonstrated on the computer and discussed.

Students were encouraged to solve differential equations that occurred in class by going to the microcomputer laboratory later. The teacher thought it worthwhile to take the whole class to the microcomputer laboratory to work in groups to investigate graphs of $y'' + ay = 0$ for positive and negative values of a and different initial values. After the lesson he felt that most groups had derived some extra insight into hyperbolic and circular functions and also into the concept of function.

Three students decided to use ANALOG as the basis for the investigation to be submitted as part of their Mode 2 examination.

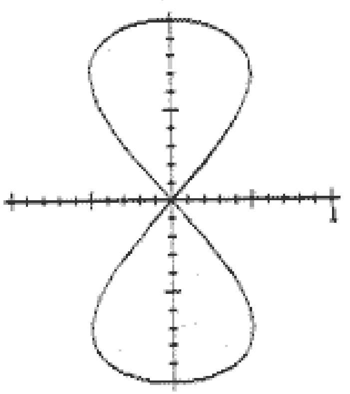
The teacher said "The more I use the program the more different ways I see of using it. I feel we are a long way from exhausting its capabilities."

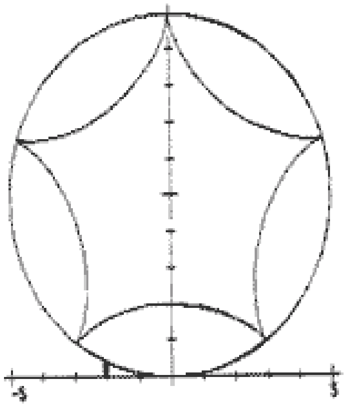
An Additional Mathematics class

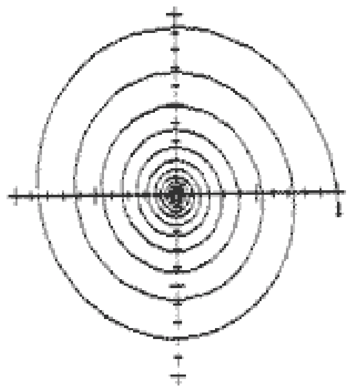
One teacher used ANALOG with an additional mathematics class in demonstration mode. He felt that students didn't really appreciate what was going on when he was discussing questions like

"A particle moves so that its position after t seconds is given by $x = 5t^3 - 8t^2 + t$. Describe the subsequent motion".

This was easily put onto the x axis and y kept fixed so that







Above and bottom left: Investigations using ANALOG

motion and position could be observed. After discussion a time base was introduced.

The teacher felt that the students gained insight into the meaning of negative position and velocity from this demonstration and intends to use ANALOG next year when the same group start A-level Maths.

Conclusion

In general, feedback from users has been positive and encouraging. When students are given a chance to use ANALOG they quite quickly became involved in a real mathematical environment and begin to see ideas from a new perspective. ■

Pat Deery

Pat Deery works at North West College of Technology, Londonderry, Northern Ireland. ANALOG is one of the programs included on the fifth *Micromath* disk, now available from the ATM Office.