

A cautionary tale

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I have known for a long time the importance of using questioning in classrooms to elicit explanations of pupils' answers so that all the class understand why an answer is correct. However, the need to move on can sometimes make it easier to accept the right answer and explain *for them*. In recent years I have twice been hit forcibly by why that isn't ever sufficient if we are truly to enable pupils to move their learning forward. In both cases the question posed was inadvertently flawed, so that pupils could make false connections.

In the first instance, the lesson was to be about trial and improvement. The teacher I was working with and I had planned a starter that would check and refresh pupils' memories about square numbers and square roots. I was also modelling the use of mini-whiteboards for the purposes of assessment. We had checked that they could remember what a square number was and that they knew the square numbers up to 100. What we wanted next was to check their estimating ability for the square roots of non-square numbers:

Give me two whole numbers that are either side of the square root of 40?

Most of the whiteboards were blank, but one said '5 and 8', and the explanation given was reasonable and included 25 and 64, but I realised the need for rephrasing, and the next question was:

Give me two consecutive whole numbers that are either side of the square root of 42.

Still a lot of blank boards – and it became clear that the word 'consecutive' was causing the problem – but one board had had '6 and 7' on it, and the explanation was about them being next to one another. I tidied the explanation up with the 36 and 49 in the rephrasing. The next question was identical, but with 56 instead of 42. A few more correct

answers on the boards – it was becoming clear that the class hadn't done estimating of square roots before. However, I picked a pupil with '7 and 8' on his board, who had previously had incorrect answers, and his explanation identified the flaw in the questions asked so far:

7 times 8 makes 56.

I realised the false trail I was creating! I hadn't noticed that the first pair of numbers had a product of 40 or that the correct answer to the second question happened also to be the product of 42. By choosing 56 for the third number, I was creating a pattern of answers for a pupil who had not understood half the previous explanations but understood a connection between the answers given, the question and his prior knowledge. With some more appropriately chosen numbers, and lots of number lines, the lesson moved on as intended.

The second incident involved more ingenuity by the pupil, and I suspect that others had followed the same false trail. It was a Y8 optional paper question on means. Part 1 had three cards with the numbers 4, 8 and 9 on them, and the question asked the pupils to explain why the mean of these three numbers was 7. Part 2 had two sets of three blank cards and asked them to find two sets of numbers with a mean of 5.

With a teacher, I was analysing the papers from one school to look for curricular targets. There were some very odd patterns in the marks in general, but a significant number of pupils had failed to get the mark for part 1 of this question but had scored one out of two marks for part 2. We realised that some (at least) of the pupils who had this pattern had lost the first mark because we had decided to disallow

$$4 + 8 + 9 = 21 \div 3 = 7,$$

but there were still some with 'rubbish' for part 1 but with one correct solution

for part 2. As a result of this, and several other strange patterns in the marks, we decided that I would interview some pupils. As part of the process, I got pupils to work as a small group on this and several other questions (on enlarged A3 sheets) and then talked to them about their working out.

Most groups couldn't do part 1, and in discussion it became clear were completely confused about the difference between mean, median, mode, range and several other bits of data-handling vocabulary. However, I was intrigued by the lad who very rapidly gave me 2, 6 and 7 for the first set of part 2 but hadn't managed part 1 or a second for part 2. His explanation was:

8 is 1 above 7, 9 is 2 above 7 and 4 is 3 below 7, so 6 is 1 above 5, 7 is 2 above 5 and 2 is 3 below 5".

It works – and there is potentially an investigation into why. I told him that although it was a solution, I would have to think about why it worked, but he hadn't used the area of mathematics the question was testing, and then we discussed the correct definition of mean. He was then able to answer the first part and find several other solutions for part 2. Maybe the original question writer realised the possibility of a link, and so asked for two answers for part 2, so that understanding 'mean' could only be demonstrated by more than half the marks for the question.

What is undoubtedly true is that misconceptions occur because pupils can draw a false conclusion from patterns accidentally laid out in front of them. Questioning unravels false logic. However, pupils also need to experience the skill of choosing the correct mathematics to solve the problem. Does that mean that the deliberate laying of false trails allows pupils to learn some lessons about appropriate use of creativity within mathematics? Certainly, as a first step, we need their explanations to check that an appropriate route to the solution has been used.

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