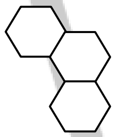
(A) Puzzles

E2) **Three gentlemen and their ties**.
Three gentlemen are in a meeting, Mr White, Mr Green and Mr Brown. They are each wearing a tie, one white, one green, one brown. Mr White says, "Did you notice the colours of our ties are different from the names of those wearing them?". The person who is wearing the green tie says "Yes, you are right".
Who is wearing what?

M3). **The 'dud' note**
My friend Sid runs a fruit and vegetable stall. The other day he was cheated by someone who bought veg costing £5 with a dud £20 note, receiving £15 change.
The £5 worth of veg had cost Sid £4 in the wholesale market, so he claims he lost £1 profit, as well as £20 because the note was dud, £4 for the goods he handed over, and £15 for the change he gave, £40 in all. The thief thinks he made £15 from the change and £4 from the veg he was given. So where is the other £21?

M/H 10. Hexagon patterns (Plus 6, 1998)



Three hexagon mats can be placed edge to edge as shown.
How else can they be arranged if edges must meet edges?
Find the number of ways of arranging 4 hexagons; draw diagrams using the hexagons and also using the equivalent line patterns.
Show there are 6 patterns involving 4 hexagons, and investigate 5 hexagons.

M13. Turn-number Puzzle
The digits 0, 1, 8 look the same when they are turned upside down; 6 is changed into 9 and vice versa..
So there are four two-digit numbers which look the same turned upside down:

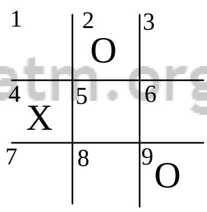
11 69 88 96

Make a list of all 3-digit "turnumbers" , then find all the 4-digit ones. Can you predict how many 5-digit and 6-digit "turnumbers" there will be?

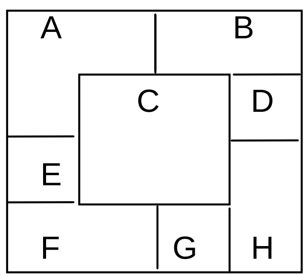
B Logic

E2 Coins problem
Three coins are placed Heads upwards. You are allowed to turn over two coins next to each other - either the two on the left (l), or the two on the right (R). No other moves are permitted. Is it possible to end up with all three coins showing tails?

E 4. Noughts and Crosses
You are X and it is your move.
You are thinking of going to square 6.
(i) Is this a good move?
(ii) Explain fully why you think so.



E5 Carpet Tiles
Square carpet tiles, all the same size, are placed in a pile on the floor as shown.
Find the order in which they are placed, finishing with C at the top.



M9 Mixed-up chocolates
Sarah wraps up 3 boxes of chocolates - plain, milk and mixed. Her younger brother mixes up the labels in such a way that no box has the correct label on it.
Can I discover which box is which by opening just <u>one</u> parcel?

M10 Marbles
4 jars contain 20, 19, 13 and 10 marbles.
You are allowed to take 1 marble from each of 3 jars and put the 3 in the fourth jar.
If you continue, can you ever get all the marbles in one jar?

C Conjectures

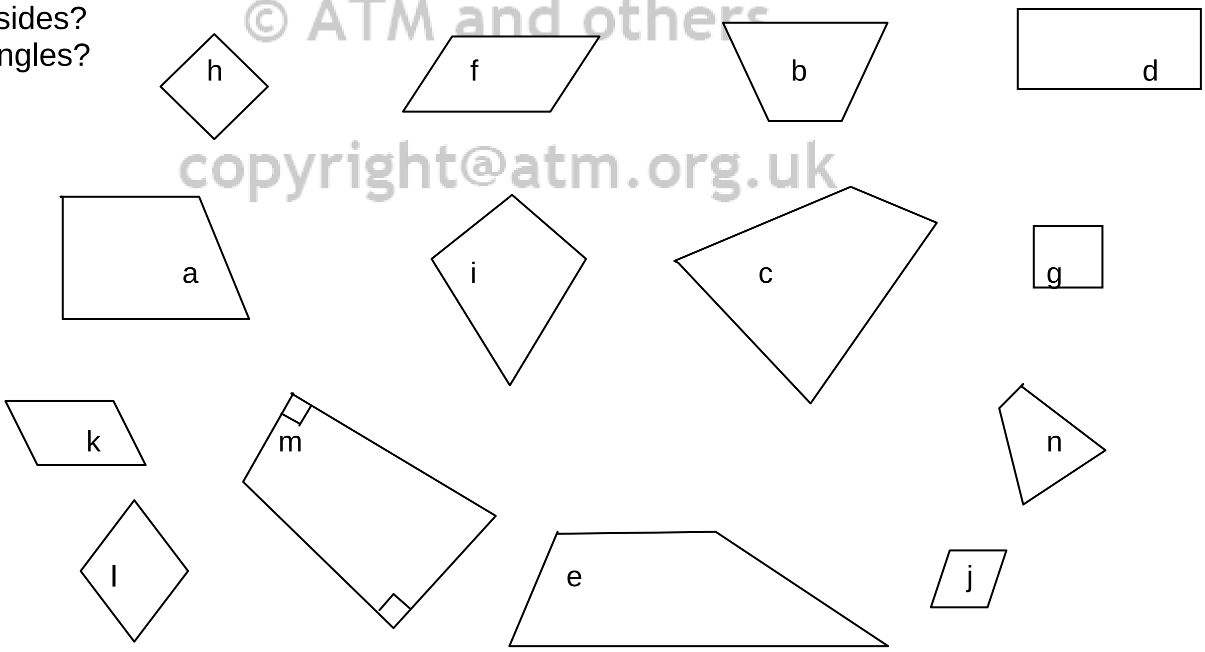
2. Stamps 1, 2, 3
I have a supply of stamps, of value 1, 2, 3 units. In how many ways can I combine them to make up a sum of 4 units (disregarding order)?
How many ways are there of making up 3 units, 2 units, 1 unit? ... ?
Try 5 units. What do you think the result will be for 6 units? Check!
Now try 7 units…..

3. Betty's Conjectures
a) Choose any whole number. Multiply by the next whole number. Explain why the answer is always even
b) Now add on 11. Explain why the answer is always a prime number.
c) Explain why the differences increase by 2 (do they?)

5. Beads of 2 colours
I have a supply of blue and green beads.
How many different ways are there of arranging 3 of them in order? [BGG counts as different from GBG].
Sam thinks the answer is 3x3 = 9, but we can only find 8 so far.
Sam argues that with 2 beads the answer is 4 (4=2x2) and with 1 bead the answer will be 1x1=1,
He suggests we try 4 beads.
What do you think?

D Geometry

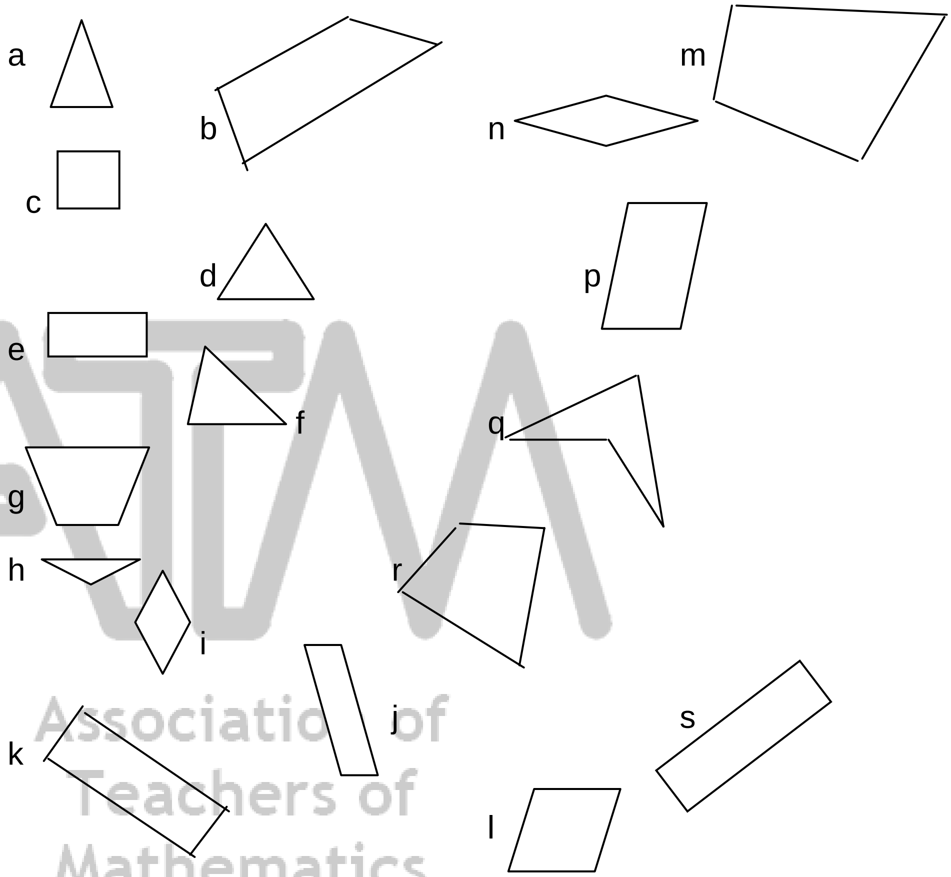
1.M Classifying quadrilaterals
Four sided shapes are called quadrilaterals.
Can you sort these into different types? Can you give names to any of them?
Some ideas for sorting - which have:
- parallel sides?
- equal sides?
- right angles?



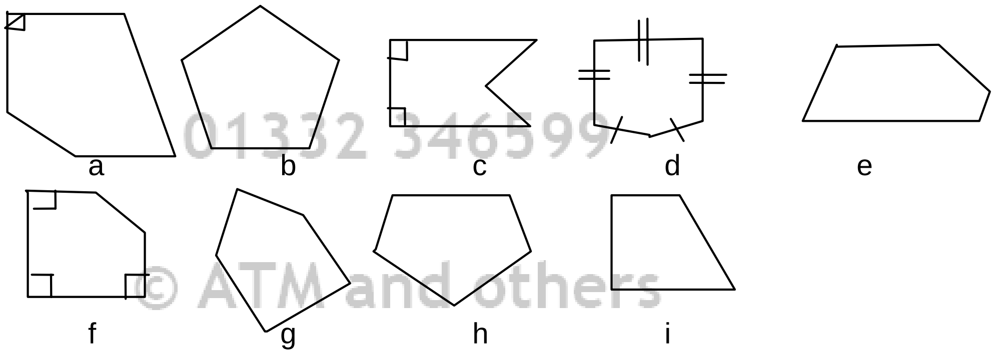
2. E Quadrilateral (and triangle) Quiz

Match each name with the corresponding letter(s):

1. rectangle
2. rhombus
3. isosceles trapezium
4. irregular quadrilateral
5. square
6. irregular trapezium
7. parallelogram
8. kite
9. equilateral triangle
10. isosceles triangle
11. scalene triangle



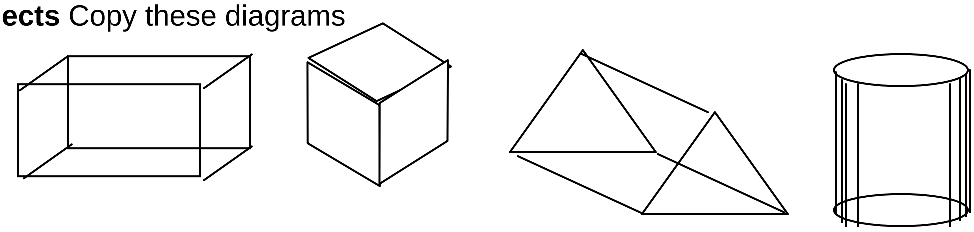
4. Divide these pentagons into groups;
give reasons:



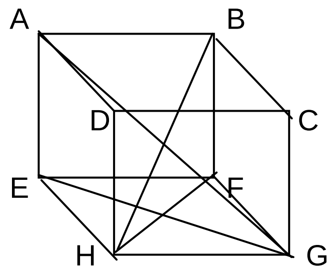
10 M Cube

A cube of dimensions 4x4x4 is made up of unit cubes and is painted red on the outside.
How many of the unit cubes are red on:
3 sides 2 sides 1 side 0 sides?
If the cube is cut up by three plane cuts, is it possible to reassemble the pieces (by glueing them together) so that no red is showing?

11E Drawing 3D objects Copy these diagrams

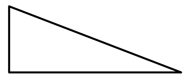


Box:



Is the angle EHG equal to angle FGH?
(even though they may not look it)
Are any of the triangles right angled?
If so, where are the right angles?
What is the shortest path from A to G on the surface of the box across the edge DH?
(Draw the faces ADHE and DCGH opened out)
If HD = 4, DC = 3, draw rectangle CDHG and find HC. If also BC = 12, draw rectangle HCBE and find length of diagonal CE.
Is it the same as length BH? And as AG?

12 M Quadrilaterals
Make as many different quadrilaterals as you can with 4 of these triangles



E Calculators

1. Getting to know your calculator
A good method of learning how to operate a calculator is to try out something very simple - to "suck it and see". So you could do 3x4 and see if it gives what you expect (12)
a) Try 24 ÷ 8 - it should give 3.
Try a few more - make up your own numbers. Predict first what you will get - then check.

b) Now try 2+6x5 it will probably give 32, not 40. This is because it does the multiplication **before** the addition.
Try 5x6+2 and 5x(6+2)
What about these (say what you think the answer will be, then check:
5x(2+6) and (5x2)+6 and 5x2+6.

3. Arithmetic made easy: (1SF arithmetic)

In 1SF arithmetic **you are only allowed to use 1 digit**, together with as many zeros as you need (to show which are H, T, U - ie. where the decimal point is).

b) How to use 1SF arithmetic to make life easier!
Suppose you want to know (roughly) the answer to 12.2 + 43.7X4.6.
Write each number in 1SF arithmetic so we get 10 + 40x5 which is 210
(or 200 in 1SF arithmetic). So the (rough) answer is 200 (The exact answer is 213.22).
This is useful in checking for mistakes in using the calculator - it is very easy to enter + instead of x, or to misplace the decimal point eg 356 instead of 3.56 .

c) Tracy and Sharon were learning to use a calculator. They kept getting the wrong answers by pressing the wrong keys! Can you help?
Example "Work out 3x6" T got 18, S got 9. Who was right? What had the other one done (probably)?
Here are some more examples. Say who was correct, and what the other one did wrong. (Sometimes both are wrong!!)

i) Q 12+5	S 17	T 7
ii) Q 100÷2	S 98	T 50
iii) Q 8x4	S 2	T 56
iv) Q 7+5	S 75	T1.4

Make up some more wrong answers for your partner. See if s/he can spot how they were obtained.

f) An alternative way - 'cancelling' with very rough approximations.

$$\frac{365 \times 42}{27 \times 13} = \frac{365 \times 42}{27 \times 13} = \frac{10 \times 3}{1 \times 1} = 30$$

(We 'cancel' 13 into 42 giving 3 (approx.) and 27 into 365 giving 10, (**very** approx). Since both approximations give too small an answer, our estimate is too small, and so we know the answer is greater than 30) [In fact, the answer is 43.67]

4 Estimation (I) (Modelling)

1. How many seconds in your life so far?
10 years = 10 x 365 x 24 x 60 x 60 seconds = 3×10^8, approx (300000000)

5. There are 800 million poor people in India. How long would it take for them to walk past you at 1 per second?
Time = 800000000/60 seconds = 800000000/60/60/24 days = 800000000/60/60/24/365 years = 25 years. (approx)

e) Estimation game - Rules
Estimate the answer using one figure plus as many zeros as required.
If your estimate is less than twice the correct answer and more than half the correct answer, you gain a point. (You work out the correct answer using a calculator).
Example:

$\frac{18.92 \times 7.064}{56.7 \times 0.888}$ "=" $\frac{20 \times 10}{50 \times 1}$ = 4 | $\frac{0.9 \times 17}{10.7 \times 0.5}$ = $\frac{0.9 \times 17}{0.5 \times 10.7}$ "=" 2 x 2 = 4

7. Large Numbers

c) Problem
By arranging three '2's and other symbols like +, -, x, ÷, √, what other numbers can you make? (all three '2's must be used)
eg (2+2)/2 = 2, 22/2 = 11, …..
What is the biggest you can get?

MBII Calculators- p (extracts)

1. Getting to know your calculator

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a) Try 24 ÷ 8 - it should give 3.

Try a few more - make up your own numbers. Predict first what you will get - then check.

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Try 5x6+2 and 5x(6+2)

What about these (say what you think the answer will be, then check:

5x(2+6) and (5x2)+6 and 5x2+6.

c) Investigate the 'square' key (the one that is labelled x^2):

Try 3 x^2 5 x^2 10 x^2 6 x^2 100 x^2

2. Use your calculator to help you in these:

a). Underline the biggest number in i) 6.271 , 6.4 ii) 3.79, 3.42

iii) 4.029, 4.91 iv) 8.75, 8.26

How can the calculator help??

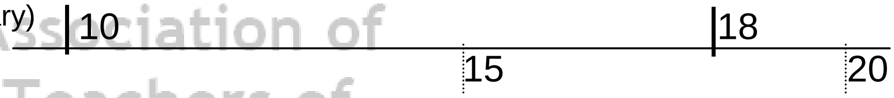
(Now make up some more examples of your own)

d). How to catch a number in a "trap":

Problem: - 2.7 x 5.3

i) This is more than 2 x 5 (lower boundary)

It is less than 3 x 6 (upper boundary)



ii)Mark each of these on a line (You have 'trapped' the answer between 10 and 18)

iii) Now work out 2.7x5.3 with your calculator and mark the answer on the line. (It ought to lie between the two boundary marks).

iv) Do the same for these 6.3x4.2 7.5x3.8, 7.3 x 11.8

Make up some more.

3. Arithmetic made easy!

(1SF arithmetic)

In 1SF arithmetic **you are only allowed to use 1 digit**, together with as many zeros as you need (to show which are H, T, U - ie. where the decimal point is).

So you can't write 4.72 - the nearest whole number is 5.

Instead of 472 you would write 500 (why not 400?)

a) How would you write 33? 37? 402? 0.74?

For 0.32 you would write 0.3, for 0.37 you would write 0.4, and for 0.425 you would write ____?

b) How to use 1SF arithmetic to make life easier!

Suppose you want to know (roughly) the answer to 12.2 + 43.7X4.6.

Write each number in 1SF arithmetic so we get 10 + 40x5 which is 210 (or 200 in 1SF arithmetic). So the (rough) answer is 200 (The exact answer is 213.22).

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h) A 'MONSTER'

Estimate the value of $$\frac{76.03 \times 908 \times 101.2 \times 6.317}{161.3 \times 4.03 \times 49 \times 1.26}$$

Make up 2 monsters of your own, estimate and then check.

4 Estimation (I) (Modelling)

1. How many seconds in your life so far?
10 years = 10 x 365 x 24 x 60 x 60 seconds = 3×10^8, approx (300000000)

2. How many bricks in this school
School is 100 x 200 yards, with 4 bricks per yard.
There are 30 bricks per storey, and 2 storeys
Total bricks = 100x200x4x30x2 = 4800000, say 5 million

5. There are 800 million poor people in India. How long would it take for them to walk past you at 1 per second?
Time = 800000000/60 seconds = 800000000/60/60/24 days = 800000000/60/60/24/365 years = 25 years. (approx)

5. Estimation(II) (Approximation)

a) Estimate (Then check by calculator):

27.3x5.2 2.7x3.3
6.2 x8.1 100/3.8
7.2/1.9
17.3 +18.2 + 23.1 + 19.07 + 25 + 16.33

Make up one for a partner.
[Her answer must be **more than half and less than twice** the correct one to count as right]

e) Estimation game - Rules

Estimate the answer using one figure plus as many zeros as required.
If your estimate is less than twice the correct answer and more than half the correct answer, you gain a point. (You work out the correct answer using a calculator).

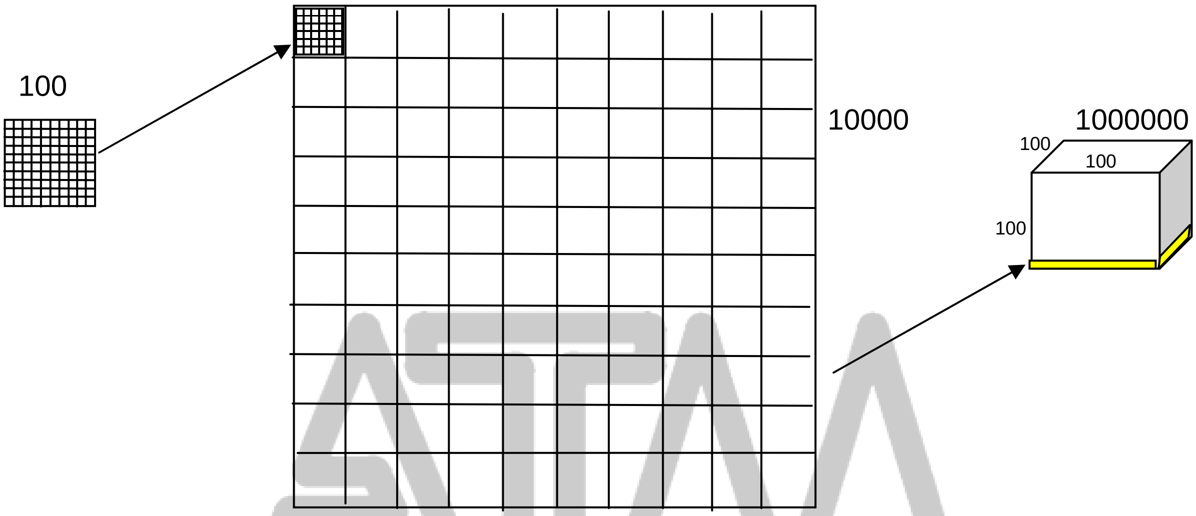
Example:

$$\frac{18.92 \times 7.064}{56.7 \times 0.888} \text{ "=" } \frac{20 \times 10}{50 \times 1} = 4 \qquad \bigg| \qquad \frac{0.9 \times 17}{10.7 \times 0.5} = \frac{0.9 \times 17}{0.5 \times 10.7} \text{ "=" } 2 \times 2 = 4$$

7 Large numbers

a) How big is a million?
A '100 square' is 10 x 10. What would a Million square look like?

How could you make a Million cube?



How long is a million seconds?
Using a metre ruler
This is 100cm or 1000 millimetres

b) Largest number on my calculator is 10^{99}
That is 10000000…….(99 zeros) ….0000
How long to count this, at 1 per second?

c) Problem
By arranging three '2's and other symbols like +, -, x, ÷, √, what other numbers can you make? (all three '2's must be used)
eg (2+2)/2 = 2, 22/2 = 11, …..
What is the biggest you can get?

How to get very large numbers
Reminder: 2^{10} is 1024 which is about $1000=10^3$
Introduction:

Note: 5^{2^3} is read as (a) $5^{(2^3)}$ NOT (b) $(5^2)^3$

(a) is 5^8 = 5x5x5x5x5x5x5x5 (b) is 25^3 = 25x25x25 = 5x5x5x5x5x5 = 5^6

So

$$2^{2^2} = 2^4 = 16 \text{ which happens to be the same as } (2^2)^2 = 4^2 = 16$$

BUT $2^{(2^{2^2})} = 2^{16} \approx 64000$ while $\left(2^{2^2}\right)^2 = 16^2 = 256 = 2^8$

So, with four '2's we can achieve around 64000, but since $2^{22} \approx 4\text{x}10^6$ = 4M, we can do better :

$$2^{2^{22}} \approx 2^{4M} = 16^M > 10^M$$

With five '2's :

$$2^{2^{2^{22}}} \approx 2^{10^M} = \text{X, say}$$

X works out at 10^K, where K is (roughly) 1 followed by (one million)/3 zeros.

X is the number 1 followed by K zeros and is unimaginably big!!!

[K itself would take about 4 days to write, since there are about 86000 (say 100000) seconds in a day]

K = 100000…………(one million/3 zeros)………….0.

K is the number of zeros you have to write if you are writing X !!!!

For comparison, $$2^{2^{2^{2^{2}}}} \approx 2^{64000} \approx 10^{19200}$$

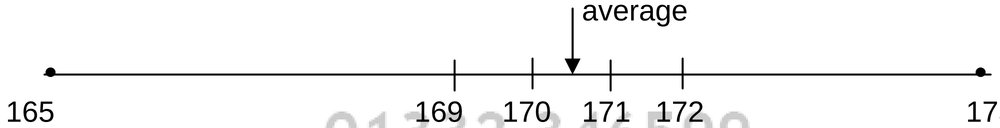
10^{19200} is 1 followed by 19200 zeros.
(If I write one zero per second it would take me about 5 hours just to **write** this number, and it would need about 4 pages of very small writing!!) [3600 sec per hour]
[5000 per page x 4 pages = 20000]

8 Drawing and measurement

a) Draw a triangle with sides 8cm and 15 cm and a right angle between them. (Use a ruler to get the sides as exactly as you can, but 'guess' the right angle.)
Now measure the hypotenuse (in millimetres).
Will, James, Charlie, Edward and Fred got these results:
W 171, J 172, C 169 E 170 F 235
How can we find the 'correct' answer?

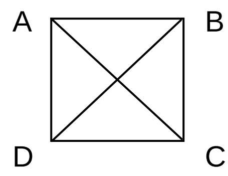
It looks as though Fred has made a mistake - so, for the moment, we will disregard his answer.
The average of the other measurements is (171+172+169+170)/4 = 170.5
(Can you think of a quicker way of finding the average?)

We can show these results in a diagram, showing also the average value:

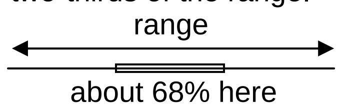


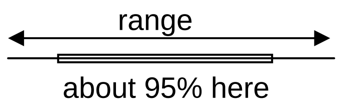
The true answer is 170 - the average is close to the true value. [Fred got 169.5 on his second attempt, (which would bring the average down to 170.3)]

Do the same exercise with the following measurements:
a) Sides 20cm 21cm, angle 90 degrees b) sides 25cm, 7 cm, angle 90 degrees
c) sides 24cm, 7 cm, angle 90 degrees
The true answer can be found using the theorem of Pythagoras

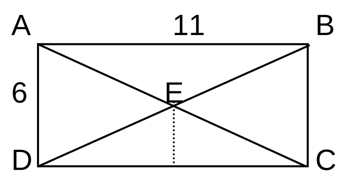


b) If you draw a diagram and measure lengths which seem to be nearly the same, how can you tell whether they are really different , or whether they are 'meant' to be equal, and differ only because of errors in measuring?
(Example - in a square, the diagonals AC and BD **should** be equal - but when you measure them, the answers may be **slightly** different).
How to find out
The difference between the largest and smallest measurements is called the **range.**
In many situations (heights, weights, factory components, scale measurements…) about 68% of the measurements are concentrated in the middle third of the range, and about 95% in the middle two-thirds of the range.

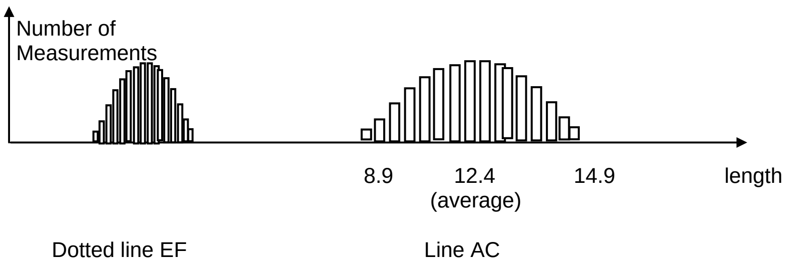




Note: This isn't **always true**, but it happens a lot.
c) Example:



A class of children draw the rectangle ABCD as shown
and measure the lengths of AC and EF.
Their results are plotted on a bar-graph like that below.
How do we find the true values of AC and EF?



From the measurements for AC, we see the range is about 8.9 -14.9 ie. 6 and the average is 12.5.
So the middle third (where 68% of the results are likely to be) is from 11.4 to 13.4
This suggests that i) the true answer is near 12.4
and ii) there is about a 2/3 chance that the true value lies between 11.4 and 13.4

Draw the rectangle and make the measurements for AC and EF. Pool results with other pupils and estimate the true values of AC and EF.
[Actually, we know the true value of EF - what is it?]

B2 Two Number Patterns

(a) Work out (1 × 2 × 3 × 4) + 1, (2 × 3 × 4 × 5)+1, (3 × 4 × 5 × 6)+ 1.

Can you state a rule? Can you prove it?

(b) Work out
$1^3 + 2^3$

$1^3 + 2^3 + 3^3$

$1^3 + 2^3 + 3^3 + 4^3$

In each case the result is a square number. Can you predict the next result? Try it.
Can you state a general formula? (Hint: triangular numbers may help you)

B3 Sums of Squares

Some numbers can be written as the sum of two squares $N = x^2 + y^2$.
17 can, ($17 = 4^2 + 1^2$), so can 13, ($13 = 3^2 + 2^2$).
19 cannot. (Try it).

Make a list of all the prime numbers (≥ 3) up to 53, and try to express each of them in this way. Classify them into two sets, according to whether they *can* or *cannot* be written as the sum of two squares. Can you find a rule for deciding which set a prime number should be placed in? Test your rule on a prime greater than 53.

B6 Patterns, conjectures and proofs - 3 problems

(a) Investigate the "theorems":
(i) $n^3 - n$ is always a multiple of 3
(ii) $n^5 - n$ is always a multiple of 5
(iii) $n^7 - n$ is always a multiple of 7
(iv) $n^9 - n$ is always a multiple of 9
Generalise.

(b) Evaluate the expression $x^2 + x + 41$ for x = 0, 1, 2, 3....
Is the result *always* prime?

(c) Make a table of squares (cubes, fourth powers) of the numbers from 1 - 20.
What patterns occur in the table and how may they be explained?

B8 Sums of digits

(a)

(i) Which of the following numbers are divisible by 3 ?

879, 3762, 28795, 7826, 6134, 3213, 9859, 7290

(ii) Which of the numbers in (i) are divisible by 9?

(iii) Make a table as follows

Number	Sum of digits	Div by 3?	Div by 9?
879	8+7+9 = 24	yes	no
3762	18	yes	yes

Can you see any connection between divisibility by 3 or 9 and the sum of the digits?

(b) A problem: Start by choosing any whole number over 100 but less than 1000 (that is, a number of three digits) Call it N, Add up the three digits of your number N, and subtract the answer from your original number. This gives you a new number - call it M. Add up the digits of your new number M. The answer is often 9. Try choosing some other starting numbers for N to see if you can get a different answer at the end. See what other answers you can find. Now try this with numbers which have four digits, five digits, etc.

B10 Number tricks

(a) Choose any number between 1 and 9 inclusive. Multiply it by 10 and add 1. Then multiply your answer by 10 and add 7.
Try this again, starting with a different number. What do you notice ? Try some more numbers to see if your idea works every time. (You can pretend to be a mind reader!) Can you see why this trick always works? Make up another trick like this one.

(b) Think of a number between 1 and 10. Multiply it by 4, add 5, multiply by 5 and add 4, and finally multiply by 5. Subtract 145. Try this with other starting numbers. What do you notice? Can you explain it? Make up another similar trick.

(c) Think of a single digit number, (that is one between 1 and 10), double it, add 10 and double the answer. Subtract 10 and halve the result. Now take away twice your original number. What is your answer? Try this with another starting number. Do you notice anything about your answer ?
Try to explain how this trick works and make up another one like it.

(d) Another well-known trick is this:
Take any three digit number, in which the units digit and the hundreds digit differ by at least 2. Reverse the digits of your number, and subtract the smaller of these two numbers from the larger. This give s you a new number, N. Reverse the digits of N, and add the answer to N. You will get 1089 whatever number you start with. Try it:
[Example: I choose 365. Then 563 - 365 = 198. 198 + 891 = 1089]

** To explain why this happens is not too easy - see if *you* can.