Successful Strategies for Mental Arithmetic

Observations in the classroom

It is clear that some people have quick ways to work out calculations whereas others have to do a laborious amount of working out before they get the answer. 'Issues in teaching numeracy in primary schools' edited by Ian Thompson quotes research suggesting that

" The children who fail in mathematics are actually carrying out a more difficult kind of mathematics than those who succeed. He found a clear tendency with low-attaining pupils to use primitive mathematical objects and primary processes, for example counting to solve addition problems, or repeated addition to solve problems which could be more effectively solved using multiplication."

I have seen this so many times with children who fall behind their age group in mathematics.

Since the introduction of the National Numeracy Strategy there has been an emphasis on mental calculation as this is considered to develop a sense of number and place value. In my research I looked at whether children can be taught particular mental strategies as short cuts to calculating certain problems. Then, having been taught the strategies, I looked at whether the children adopted the strategies as part of their tools for mental calculation.

The mental calculation strategies

I chose two mental strategies which I thought my students would not have met before and which I thought would intrigue them. I found these two as part of a suite of methods called Vedic maths and there is a website at www.vedicmaths.org which has a tutorial on how to use these methods. Vedic maths is supposed to be an ancient way of doing maths devised in India which is in tune with how the mind works.

The first method is one which I had met before and have found useful. It is a way of subtracting numbers from 1, 10, 100, 1000, etc. The Vedic mantra says 'all from 9 and the last from 10' which I changed to ' make each number up to 9 and the last one up to 10'.

For example 1000-345=655
3 up to 9 needs 6
4 up to 9 needs 5
5 up to 10 needs 5

The second method is a way of mentally multiplying two two-digit numbers. It also provides a very nice way of multiplying on paper in one line. The Vedic mantra says 'vertical and crosswise'.

For example
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 32
 21x
672
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The first column gives 3x2=6
Multiplying crosswise gives 3x1+2x2=7
The second column gives 2x1=2

This method is much more complicated particularly when carrying figures are involved and requires the student to hold figures in their head. For this reason I only used it with top set classes.

The experiment
Roger Merry, a senior lecturer at Leicester University school of Education was my mentor during this research. He helped me to devise the experiments and gave me advice on how to set up the target and control groups. We met regularly, usually once a month, to talk about how the research was progressing.
During the period of my research I changed schools so I was able to test the methods with two school populations. I used the subtraction method with mixed ability year 7 classes and year 9 classes from parallel sets in two half year groups. I used the multiplication method with parallel year 8 classes in one school and parallel year 7 classes in the next school. So there were always target classes (who were shown the method) and control classes (who were not shown the method) which were matched academically.
In each case the classes were given a test before the method was taught. I then interviewed some of the children who did well in the tests. The target groups were then taught the method for at least 2 weeks for 10 minutes at the end of lessons.. After this all the groups were tested again and then a third test was given after a month to 3 months during which time the method was not mentioned . I then interviewed some of the children who had made significant progress in the tests.

The subtraction method
I introduced this method as a quick way to do subtraction when you have 1 followed by several zeroes. I introduced the mantra ‘up to 9 and the last one up to 10’ and went through the method using examples where we were subtracting from positive powers of 10. I deliberately did not use examples where the we subtracted money from a whole number of pounds or where we subtracted decimals less than 1 away from 1.

Eg. £10.00-£3.46 or 1-0.34
The reason for this is that I wanted to see whether the students would be able to adapt the method to slightly different problems. The test of 20 questions is below

1	100-36=
2	100-51=
3	1000-777=

4	1000-283=
5	1000-505=
6	10000-2345=
7	10000-9756=
8	1000-57=
9	1000-38=
10	10000-352=
11	£1 take away 42 pence=
12	£1 take away 89 pence=
13	£5 take away £1.46=
14	£5 take away £3.72 =
15	£10 take away £3.69=
16	1-0.6=
17	1-0.55=
18	1-0.95=
19	1-0.432=
20	2-0.64=

The target groups were given practice with the method for 10 minutes at the end of a lesson. In each target group there was at least one child who asked why the method works. In reply I put up an example of the standard algorithm on the board eg.

1000
 283-
1 in the thousands column becomes 10 hundreds, 1 hundred goes to make 10 tens, 1 ten goes to make 10 units,
so we end up taking 200 from 900 (2 up to 9)
80 from 90 (8 up to 9)
and 3 from 10.

The findings

After the first test, before I showed the class the method, I interviewed some students who did well in the test to see what method they used. In the mixed ability year 7 groups it tended to be the more able students who had a workable method of their own. In the year 9 groups who were in a 3rd set out of 4 it tended to be the boys who had a workable method of their own. It begs the question of whether these boys were under-achievers and ought to have been in a higher set.

Some of the children had a method very similar to the ‘up to 9 and the last one up to 10’.

Balraj 7jPCL 100-472

“work out these two(ie.hundreds and tens) by adding up to 9 and adding last number up to ten. The answer is 528”

When asked to work out 5000-418 he was able to give the answer 4582 because, he said he took the 5 down to a 4.

Philip 7jPCL 1000-476 = 524

“take 1 off the first two digits, 4 from 0 is 6, then take 1
7 from next one (ie. Next zero), then take 1
then 6 from it (last zero) is 4 and you don’t need to take the 1 from that”

Philip’s method is similar to Balraj’s method in that he deals with the digits separately. He simply takes the 1 from the subtraction rather than taking the 1 from the 10. This is akin to the old style algorithm where you ‘Pay back 1’ on the bottom line as opposed to Balraj’s method which is more akin to the new style algorithm where you change the one thousand to 10 hundreds and one of the hundreds becomes 10 tens, etc.

Neha 7jPCL 1000-471=529

“I put the number underneath in my head. When carrying, take off 1 becomes 9. First two numbers become 9 and last becomes 10. So take last digit from 10 and other two from 9”

She seems to have done this problem by visualising the commonly used algorithm.

Kyle 7jPCL 1000-471=529

“I just take 4 from 10 makes 600. Then take 70 and that makes 530 and then I take 1 and that equals 529”

Kyle has done this by breaking down 471 into 400, 70 and 1 and subtracting them off in turn.

These children were also able to manage the decimal problems by converting the problem into whole numbers and then adjusting afterwards eg. 1-0.3 pretend that’s 10-3
1-0.27 pretend that’s 100-27.

These very able children at the top of a mixed ability group seem to have solved the problems with reference to what happens in the standard algorithm so it is possible that their confidence in using this algorithm helped their mental skills as well.

The students in the year 9 group tended to solve these problems by either breaking down the numbers into parts as Kyle did or by rounding up the numbers and then compensating.

Mohammed 9EJK 100-51
"go up to 60
add 60 up to 100 which gives 40
then add 9 on."

1000-38
"go up to 100
100+900=1000
then add 2 on is 40
40 +60 =100
take 2 (except he should have added 2 to compensate)

£5-£3.72
"went up to £4
£1 there
went up to 80
added 8 on

He could not start to do 1-0.55 and I think this is because he did not really understand decimal numbers.

Several of the year 9 students seemed to have quick recall of the complements of numbers up to 100. For example, when I asked Emma to work out 1000-38 she did it by taking off 100 and adding back 62. She could not explain how she knows that 100-38 is 62. Martin, who I interviewed after the method had been taught and after the second test, explained that he worked out 100-264 by taking 300 away from 1000 and then adding back on the 'other half' and he got the correct answer of 736.

<u>The statistics</u>

Roger used a non parametric test (sign test) to analyse the results. We found that, in the year 7 target group, which I had taught,

between test 1 and 2, 17 students improved
5 students declined
2 students stayed the same

this is significant at the 5% level.

However, between test 1 and 3 17 students improved
8 students declined

This is not significant at the 5% level so the effect had worn off. There were 4 students who improved then declined and their year 7 test results showed their attainment in maths to be average or below and perhaps they needed more practice with the method.

The control group that I taught showed no significant improvement at the 5% level.

I got a shock when we analysed the results of those groups taught by my colleague as their target group showed no significant improvement but the control group did show a significant improvement at the 5% level. I am not sure why my colleague's control group made such improvement! There is evidence to suggest that, for a teaching method to work, the teacher must be enthusiastic about the method.

With the year 9 groups, I tested them at the end of January and then at the beginning of April. In between I practised the subtraction method with the target group approximately 2 times a week for 10 minutes at the end of each lesson. After the second test we found that the target group had made a significant improvement at the 1% level. The control group made no significant improvement. We looked more closely at these results. Firstly, I asked the students in the target group to write on the bottom of the second test paper if they had used the method all of the time, some of the time or none of the time.

10 students reported using the method all of the time and of these 9 improved and 1 stayed the same
Their mean score went up from 8.0 to 13.9 which was a 74% increase

7 students used the method sometimes and of these 4 improved, 1 stayed the same and 2 decreased
Their mean score went up from 6.9 to 9.1 which was a 32% increase.

5 students said they did not use the method and of these 4 improved and 1 decreased.
Their mean score went up from 10.8 to 12.1 which is a 12% increase. 3 out of 5 of these students were high scorers in the tests anyway who could only make a small increase. There would be little motivation to change strategy.

Overall this showed that those students who did use the strategy had the biggest improvement in their test scores.

We also looked at how the classes did in the two halves of the test. The first half of the test deals with whole number subtractions using large numbers and I had practised these type of problems with the target group. The second half of the test deals with money and decimals and I did not practise any of these types of problems using the method. However we had spent some time in class on decimal place value and decimals on the number line. I had the feeling, when I marked the tests that both classes had made improvements on the last 10 questions. When we looked at the results of the sign tests we could see that the target group did not significantly improve on the money and decimals questions and that most of their improvements were accounted for on the first 10 questions. However the control group, who did not make any significant improvements in the test overall had improved on the money and decimals part at the 5% level. I wonder at the reasons for this and imagine that the number line and place value work we did must have

benefited the control group as it gave them a deeper understanding of decimals. However this did not work for the target group but then they had been taught by many different teachers in the 4 terms before I took them on. This is unsettling for any group of children and may have a negative effect on their learning.
The idea that understanding the numbers and having a feel for their size and position needs more study than I have done here. I later asked Cally and Sarah from the year 9 target group how much they could remember of the method after we had left it alone for 3 months. They each claimed to have forgotten how to use it but had no trouble working out subtractions from 1000. Higher than that and they had trouble even when I reminded them of the method. However they had both improved on subtracting from 1000 between the very first test and when I spoke to them so something must have happened! I am of the opinion that most students have trouble with calculations when they have no real experience of the size of the number. It would therefore be immensely valuable to spend time helping them appreciate numbers and their relative sizes.

I feel that this method of subtraction is a worthwhile method to use with most students. There are lots of situations where it is useful to subtract from a number with zeroes, for instance in money calculations and in probability problems where decimals are subtracted from 1. However it is only going to become part of a student's mental tools if it is used and practised from tome to time. I also think that it is necessary for the student to be able to visualise the number and to have an idea of its size in order to feel confident in the result of a calculation.

<u>The Multiplication Method</u>
I first gave the target and control groups the test, which is below, with the instructions that the students were to work the questions out in their heads and only write down the answer. I told them that they would expect to find this difficult and that it did not matter if they did not finish the test in the 10 minutes.

21 23x	22 13x	61 31x	14 21x
32 21x	15 13x	14 13x	16 12x
13 42x	23 31x	14 41x	28 51x
24 32x	21 47x	32 53x	12 18x
18 16x	33 44x	34 23x	26 32x

These tests were given to top set classes. I was disappointed to find that some of the students had disregarded my instructions to do all the calculating mentally and had done written working. Often this working was done in pencil and then rubbed out although I could see the indents on the paper. A few students had worked out the answer using one line of working and written down the odd carrying figure. Although this was disappointing, and put me in a quandary as to whether I should not include the results of those students, it did make me think about their reactions to possible failure. Even though I had stressed that it did not matter if they could not do the test some students still did not want to fail and so had cheated. I would much have preferred for them to not cheat and get a low mark. I wonder whether able students are not used to failing and so were bothered by the test. I did nut encounter this problem with the subtraction method which was easier to do anyway. On the other hand the less able students who found the subtraction test hard just gave up on it and did not attempt to do it by cheating. Perhaps they were more used to failing. It does bring up an aspect of learning which is that it should not be seen as shameful to fail at something. More able students may come into difficulties later if they cannot handle failure sometimes.
Another problem I encountered with the year 8 control group is that their second test was supervised by a supply teacher who did not follow instructions. Instead of 10 minutes this person gave the class as long as they liked and I got back the tests all completed with different finishing times written on them. I cannot be sure of the test conditions either. I decided to ignore this test but, as the same test was given 3 times, it is not unlikely that the class remembered some of the questions from before. It is a problem when some of the conditions are out of the experimenter's control.

<u>The results</u>

With the year 8 target group there was a significant improvement at the 1% level between the 1^{st} and 2^{nd} tests. I did not mention the method after the 2^{nd} test but when I tested the class a 3^{rd} time the improvement remained: significant improvement at the 1% level between the 1^{st} and 3^{rd} tests. After this I interviewed a number of the students and they tended to be able to remember the method when reminded of it and thought it was a good method to use . Alice said that she had never really got on with any previous methods of long multiplication but that she understood the Vedic method. She preferred to work right to left as it makes the carrying figures easier to deal with. This was something which was mentioned by children in my next school also. The V
edic method can be used left to right which helps when calculating mentally. Or it can be used right to left which is easier when writing down the answer as the children have to do in a test. I gave the children lots of practice at mental calculation where they were only allowed to say the answer and also where they were allowed to write down the answer. On the whole they preferred the right to left way because of the carrying. Although they had a good retention of the method I did feel that it

would have to be incorporated into the class's way of working if they were not to forget it eventually.
However the control group in the 8th year had some surprising results in that they improved significantly at the 1% level between the 1st and 3rd tests. I have already commented on how I mistrust the findings of this group because of the exam conditions.

In the year 7 groups at the next school, the target group made an increase that was significant at the 5% level and the mean score improved from 7.2 to 11.8 which is a 64% increase. The control groups results did not show any significant increase at the 5% level. Their average score improved from 6.9 to 8.5 which is a 23% increase. At this school there was the most cheating and we dealt with this problem by simply not including those children's scores in our results.

Overall I feel that the results show that there is improvement shown by teaching this method of multiplying 2 digit numbers. Again, as with the subtraction method, the children did want to know why it works and I went through the reasoning with them on the board. They decided that it would get very difficult if there were numbers with more than 2 digits so that it has a limited use. (More than 2 digits can be done but I think that the method then becomes unwieldy). This method is also really only suitable for children who have the ability to hold numbers in their head whilst calculating and so is more of an interesting method for the more able.